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Class 12

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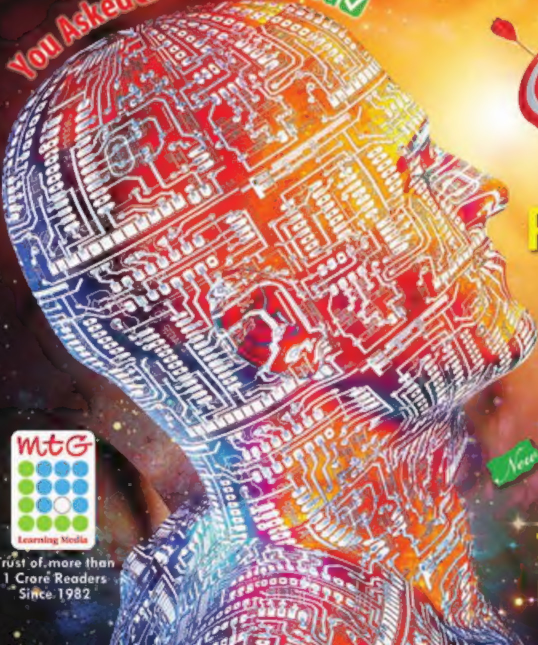

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PHYSICS for you

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Contents

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Editor : Anil Ahlawat

editorial

Expanding Borders of Education

The standard of education in India is as good as in developed nations in the west. The proof is that admissions in American colleges are based on the marks obtained in the classes from 7th to 12th exams in Indian schools. Our students are doing well in United States, France and Canada, to name only a few countries. Taking data from Delhi NCR, which is readily available, we are also a little worried.

Quo Vadis India?

In the four page supplement devoted to the colleges, their addresses and the courses available, we can say that we have made formidable advances in the spread of higher education. However, the number of colleges offering courses in B.Sc. and postgraduate studies is very small when compared to courses related to animation, management (MBA and PGDM).

Our quality students are not meant for export only. Apart from the popular courses like fashion technology, hotel management and animation, the regular science courses are offered in very few colleges. Taking any popular picture, animation plays a very large part. Flying heroes who always win and so on which the youngsters know best, every battle scene with the villans is possible only with the computer applications of animation.

Well, can we not apply them to create virtual laboratories where students can study theory and also observe the experiments? In a good laboratory, students can feel the objects, learn how to handle the apparatus, take observations without anybody imposing their conclusions. Perform experiments in the laboratory after switching off automatic calculation and studying the intensities of spectral lines on your own.The conclusions to be drawn from your experiments should be left to you. Automatic methods are good for factories but not for research workers. You have to be the master and instrument the slave.

Walk and Chalk as a professor wrote. Let the imagination of the students grow. Animation is for cinemas, not for creating scientists.

Anil Ahlawat
Editor

PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You with the suggestion of Shri Mahabir Singh. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / AIIMS / Other PMTs with additional study material.

In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / various PMTs. The detailed solutions of these problems will be published in next issue of Physics For You.

The readers who have solved five or more problems may send their solutions. The names of those who send atleast five correct solutions will be published in the next issue.

We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

By : Akhil Tewari

PROBLEM Set 10

MULTIPLE OPTION CORRECT

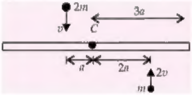
1. Standing waves are produced on a stretched string of length L with fixed ends. When there is a node at a distance $\frac{L}{3}$ from one end, then
   (a) minimum and next higher number of nodes, excluding the ends are 2, 5 respectively
   (b) minimum and next higher number of nodes, excluding the ends are 2, 4 respectively
   (c) frequency produced may be $\frac{v}{3L}$
   (d) frequency produced may be $\frac{3v}{2L}$.
   [v = Velocity of waves in the string]

2. In displacement method, the distance between object and screen is 96 cm. The ratio of length of two images formed by a convex lens placed between them is 4.84.
   (a) Ratio of the length of object to the length of shorter image is 11/5.
   (b) Distance between the two positions of the lens is 36 cm.
   (c) Focal length of the lens is 22.5 cm.
   (d) Distance of the lens from the shorter image is 30 cm.

3. When monochromatic light is incident normally on a wedge-shaped thin air film, refer figure, an interference pattern may be seen by reflection. Which of the following is/are correct ?

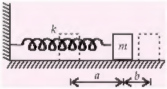
   (a) Parallel fringes are observed.
   (b) If water is introduced into the region between the plates, the fringe width decreases.
   (c) If the angle of the wedge is increased, the fringe width decreases.
   (d) When white light is used there will not be a completely dark fringe.

4. A uniform bar of length $6a$ and mass $8m$ lies on a smooth horizontal table. Two point masses m and $2m$ moving in the same horizontal plane with speeds $2v$ and v respectively, strike the bar (as shown in the figure) and stick to the bar after collision. Denoting angular velocity, total energy and velocity of centre of mass by ω, E and v_c respectively, we have after collision



   (a) $v_c = 0$
   (b) $\omega = \frac{3v}{5a}$
   (c) $\omega = \frac{v}{5a}$
   (d) $E = \frac{3mv^2}{5}$

5. The spring is compressed by a distance 'a' and released. The block again comes to rest when the spring is elongated by a distance 'b'. During this process,



(a) work done by the spring on the block $=\frac{1}{2}k(a^2+b^2)$

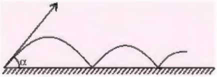
(b) work done by the spring on the block $=\frac{1}{2}k(a^2-b^2)$

(c) co-efficient of friction $=\dfrac{k(a-b)}{2mg}$

(d) co-efficient of friction $=\dfrac{k(a+b)}{2mg}$

SINGLE OPTION CORRECT

6. A particle of mass m is projected with velocity v_0 at an angle α with the horizontal. The co-efficient of restitution for any of its impact with the smooth ground is e.



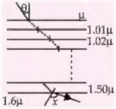
(a) Total time taken by the particle before it stops moving vertically is $\dfrac{u\sin\alpha}{g(1-e)}$

(b) Total horizontal distance moved in the time before it stops moving vertically is $\dfrac{u^2\sin2\alpha}{2g(1-e)}$

(c) Average force on the particle over the time interval in which it makes first 'n' impacts with ground equals $\dfrac{mg}{2}(1-e^n)$ directed upwards

(d) Average force on the particle over the time interval in which it makes first 'n' impact equals $\dfrac{mg}{2}(1-e^n)$ directed downwards.

7. A ray of light travelling in a medium of refractive index μ is incident at an angle θ on a composite transparent plate consisting of 50 plates of R.I. 1.01μ, 1.02μ, 1.03μ, ... , 1.50μ. The ray emerges from the composite plate into a medium of refractive index 1.6μ at angle 'x'. Then



(a) $\sin x=\left(\dfrac{1.01}{1.5}\right)^{50}\sin\theta$ (b) $\sin x=\dfrac{5}{8}\sin\theta$

(c) $\sin x=\dfrac{8}{5}\sin\theta$ (d) $\sin x=\left(\dfrac{1.5}{1.01}\right)^{50}\sin\theta$

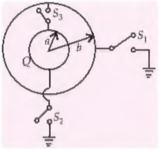
8. The molar heat capacity C for an ideal gas going through a given process is given by $C=\dfrac{a}{T}$, where 'a' is a constant. If $\gamma=\dfrac{C_P}{C_V}$, the work done by one mole of gas during heating from T_0 to ηT_0 through the given process will be

(a) $\dfrac{1}{a}\ln\eta$ (b) $a\ln\eta-\left(\dfrac{\eta-1}{\gamma-1}\right)RT_0$

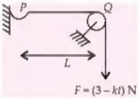
(c) $a\ln\eta-(\gamma-1)RT_0$ (d) none of these

SUBJECTIVE TYPE

9. The figure shows a conducting sphere 'A' of radius 'a' which is surrounded by a neutral conducting spherical shell B of radius 'b' ($> a$). Initially switches S_1, S_2 and S_3 are open and sphere 'A' carries a charge Q. First the switch 'S_1' is closed to connect the shell B with the ground and then opened. Now the switch 'S_2' is closed so that the sphere 'A' is grounded and then S_2 is opened. Finally, the switch 'S_3' is closed to connect the spheres together. Find the heat (in Joule) which is produced after closing the switch S_3. [Consider $b = 4$ cm, $a = 2$ cm and $Q = 8\ \mu$C]



10. In the given figure, a string of linear mass density 3×10^{-2} kg m^{-1} and length $L = 1$ m, is stretched by a force $F = (3 - kt)$ N, where 'k' is a constant and 't' is time in sec. At the time $t = 0$, a pulse is generated at the end P of the string. Find the value of k (in N s^{-1}) if the value of force becomes zero as the pulse reaches point Q.



Exam on 25 MAY

JEE FINAL TOUCH

Last 3 Years Chapterwise Questions

CLASS 12

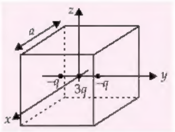
ELECTRIC CHARGES AND FIELDS

1. Two non-conducting solid spheres of radii R and $2R$, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance $2R$ from the centre of the smaller sphere, along the line joining the centres of the spheres, is zero. The ratio $\frac{\rho_1}{\rho_2}$ can be

(a) -4 (b) $-\frac{32}{25}$

(c) $\frac{32}{25}$ (d) 4 *(2013)*

2. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, $-q$ at $\left(0, \frac{-a}{4}, 0\right)$, $+3q$ at $(0, 0, 0)$ and $-q$ at $\left(0, +\frac{a}{4}, 0\right)$.



Choose the correct option(s).

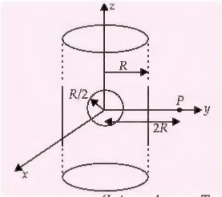
(a) The net electric flux crossing the plane $x = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = -\frac{a}{2}$.

(b) The net electric flux crossing the plane $y = +\frac{a}{2}$ is more than the net electric flux crossing the plane $y = -\frac{a}{2}$.

(c) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$.

(d) The net electric flux crossing the plane $z = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = +\frac{a}{2}$. *(2012)*

3. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $\frac{R}{2}$ with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is



(Integer Answer Type, 2012)

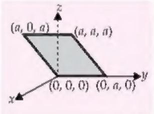
4. Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is

(a) $2E_0a^2$

(b) $\sqrt{2}E_0a^2$

(c) E_0a^2

(d) $\frac{E_0a^2}{\sqrt{2}}$



(2011)

5. A spherical metal shell A of radius R_A and a solid metal sphere B of radius R_B ($< R_A$) are kept far apart and each is given charge $+Q$. Now they are connected by a thin metal wire. Then

(a) $E_A^{\text{inside}} = 0$ (b) $Q_A > Q_B$

(c) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$

(d) $E_A^{\text{on surface}} < E_B^{\text{on surface}}$ *(2011)*

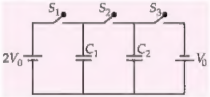
6. Four point charges, each of $+q$, are rigidly fixed at the four corners of a square planar soap film of side a. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$, where k is a constant. Then N is

(Integer Answer Type, 2011)

7. Which of the following statement(s) is/are correct?
   (a) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid.
   (b) The Gauss law can be used to calculate the field distribution around an electric dipole.
   (c) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.
   (d) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$. *(2011)*

ELECTROSTATIC POTENTIAL AND CAPACITANCE

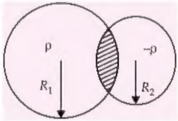
8. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time,



   (a) the charge on the upper plate of C_1 is $2CV_0$.
   (b) the charge on the upper plate of C_1 is CV_0.
   (c) the charge on the upper plate of C_2 is 0.
   (d) the charge on the upper plate of C_2 is $-CV_0$.

(2013)

9. Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively,



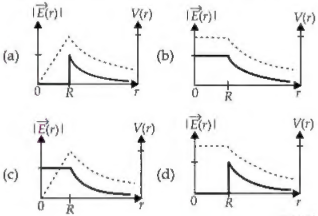
are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region,
   (a) the electrostatic field is zero.
   (b) the electrostatic potential is constant.
   (c) the electrostatic field is constant in magnitude.
   (d) the electrostatic field has same direction.

(2013)

10. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical just after release. Then X is nearly
   (a) 1×10^{-5} V
   (b) 1×10^{-7} V
   (c) 1×10^{-9} V
   (d) 1×10^{-10} V

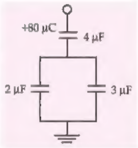
(2012)

11. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential $V(r)$ with the distance r from the centre, is best represented by which graph?



(2012)

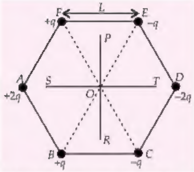
12. In the given circuit, a charge of +80 μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is



   (a) +32 μC
   (b) +40 μC
   (c) +48 μC
   (d) +80 μC

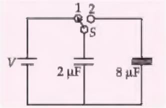
(2012)

13. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure. Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement(s) is(are) correct?



(a) The electric field at O is $6K$ along OD.
(b) The potential at O is zero.
(c) The potential at all points on the line PR is same.
(d) The potential at all points on the line ST is same. *(2012)*

14. A $2\ \mu F$ capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is

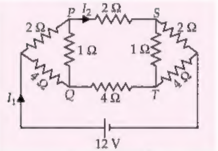


(a) 0% (b) 20%
(c) 75% (d) 80%

(2011)

CURRENT ELECTRICITY

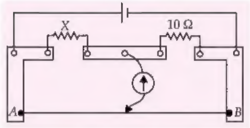
15. For the resistance network shown in the figure, choose the correct option(s).



(a) The current through PQ is zero.
(b) $I_1 = 3$ A.
(c) The potential at S is less than that at Q.
(d) $I_2 = 2$ A.

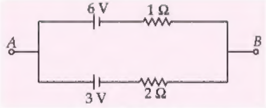
(2012)

16. A meter bridge is set-up as shown, to determine an unknown resistance X using a standard 10 ohm resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of X is



(a) 10.2 ohm (b) 10.6 ohm
(c) 10.8 ohm (d) 11.1 ohm *(2011)*

17. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volts is



(Integer Answer Type, 2011)

MOVING CHARGES AND MAGNETISM

18. A particle of mass M and positive charge Q, moving with a constant velocity $\vec{u}_1 = 4\hat{i}\ \text{m s}^{-1}$, enters a region of uniform static magnetic field normal to the x-y plane. The region of the magnetic field extends from $x = 0$ to $x = L$ for all values of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $\vec{u}_2 = 2(\sqrt{3}\hat{i} + \hat{j})\ \text{m s}^{-1}$. The correct statement(s) is (are)
(a) The direction of the magnetic field is $-z$ direction.
(b) The direction of the magnetic field is $+z$ direction.
(c) The magnitude of the magnetic field $\frac{50\pi M}{3Q}$ units.
(d) The magnitude of the magnetic field is $\frac{100\pi M}{3Q}$ units. *(2013)*

19. A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius $2R$. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement(s) is (are)
(a) In the region $0 < r < R$, the magnetic field is non-zero.
(b) In the region $R < r < 2R$, the magnetic field is along the common axis.
(c) In the region $R < r < 2R$, the magnetic field is

tangential to the circle of radius r, centered on the axis.

(d) In the region $r > 2R$, the magnetic field is non-zero. *(2013)*

Paragraph for Questions 20 and 21

A point charge Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity ω. This canbe considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$.

A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ.

20. The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change, is

(a) $-\gamma BQR^2$ (b) $-\gamma \frac{BQR^2}{2}$

(c) $\gamma \frac{BQR^2}{2}$ (d) γBQR^2

21. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is

(a) $\frac{BR}{4}$ (b) $\frac{BR}{2}$ (c) BR (d) $2BR$ *(2013)*

22. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time $t = 0$, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option(s) is (are) correct for time $t > 0$?

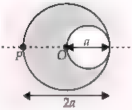
(a) If $\theta = 0°$, the charge moves in a circular path in the x-z plane.

(b) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis.

(c) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.

(d) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis *(2012)*

23. A cylindrical cavity of diameter a exists inside a cylinder of diameter $2a$ as shown in the figure. Both the cylinder and the cavity are infinitely long.



A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is

(Integer Answer Type, 2012)

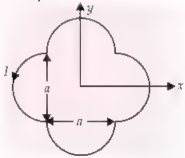
24. A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is

(a) $a^2 I\hat{k}$

(b) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$

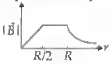
(c) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$

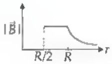
(d) $(2\pi+1)a^2 I\hat{k}$

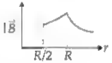


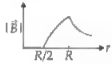
(2012)

25. An infinitely long hollow conducting cylinder with inner radius $R/2$ and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by

(a)


(b)


(c)


(d)


(2012)

26. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true?

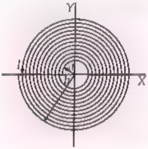
(a) They will never come out of the magnetic field region.

(b) They will come out travelling along parallel paths.

(c) They will come out at the same time.

(d) They will come out at different times. *(2011)*

27. A long insulated copper wire is closely wound as a spiral of N turns. The spiral has inner radius a and outer radius b. The spiral lies in the X-Y plane and a steady current I flows through the wire. The Z-component of the magnetic field at the centre of the spiral is



(a) $\dfrac{\mu_0 NI}{2(b-a)}\ln\left(\dfrac{b}{a}\right)$

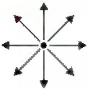
(b) $\dfrac{\mu_0 NI}{2(b-a)}\ln\left(\dfrac{b+a}{b-a}\right)$

(c) $\dfrac{\mu_0 NI}{2b}\ln\left(\dfrac{b}{a}\right)$

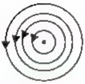
(d) $\dfrac{\mu_0 NI}{2b}\ln\left(\dfrac{b+a}{b-a}\right)$ *(2011)*

MAGNETISM AND MATTER

28. Which of the field patterns given below is valid for electric field as well as for magnetic field?

(a) 

(b) 

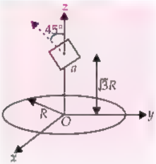
(c) 

(d) 

(2011)

ELECTROMAGNETIC INDUCTION

29. A circular wire loop of radius R is placed in the x-y plane centered at the origin O. A square loop of side a ($a << R$) having two turns is placed with its center at $z = \sqrt{3}R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\dfrac{\mu_0 a^2}{2^{p/2}R}$, then the value of p is



(Integer Answer Type, 2012)

30. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is(are)
(a) the emf induced in the loop is zero if the current is constant
(b) the emf induced in the loop is finite if the current is constant
(c) the emf induced in the loop is zero if the current decreases at a steady state
(d) the emf induced in the loop is finite if the current decreases at a steady state. *(2012)*

31. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube.

The current varies as $I = I_0\cos(300t)$ where I_0 is constant. If the magnetic moment of the loop is $N\mu_0 I_0\sin(300t)$, then N is

(Integer Answer Type, 2011)

ALTERNATING CURRENT

Paragraph for Questions 32 and 33

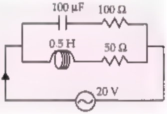
A thermal power plant produces electric power of 600 kW at 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers the dissipation is much smaller. In this method, a step-up transformer is used at the plant side so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with a power factor unity. All the currents and voltages mentioned are rms values.

32. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is

(a) 200 : 1 (b) 150 : 1 (c) 100 : 1 (d) 50 : 1

33. If the direct transmission method with a cable of resistance $0.4\ \Omega\ km^{-1}$ is used, the power dissipation (in %) during transmission is
(a) 20 (b) 30 (c) 40 (d) 50
(2013)

34. In the given circuit, the AC source has $\omega = 100$ rad/s. Considering the inductor and capacitor to be ideal, the correct choice(s) is/are



(a) the current through the circuit, I is 0.3 A.
(b) the current through the circuit, I is $0.3\sqrt{2}$ A.
(c) the voltage across 100 Ω resistor = $10\sqrt{2}$ V.
(d) the voltage across 50 Ω resistor = 10 V.
(2012)

35. A series *R-C* circuit is connected to *AC* voltage source. Consider two cases; (*A*) when *C* is without a dielectric medium and (*B*) when *C* is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?
(a) $I_R^A > I_R^B$ (b) $I_R^A < I_R^B$
(c) $V_C^A > V_C^B$ (d) $V_C^A < V_C^B$ *(2011)*

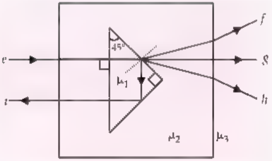
36. A series *R-C* combination is connected to an *AC* voltage of angular frequency $\omega = 500$ radian/s. If the impedance of the *R-C* circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is
(Integer Answer Type, 2011)

RAY OPTICS AND OPTICAL INSTRUMENTS

37. A ray of light travelling in the direction $\frac{1}{2}(\hat{i}+\sqrt{3}\hat{j})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{i}-\sqrt{3}\hat{j})$. The angle of incidence is
(a) 30° (b) 45°
(c) 60° (d) 75° *(2013)*

38. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is
(a) 1 m (b) 2 m (c) 3 m (d) 6 m
(2013)

39. A right angled prism of refractive index μ_1 is placed in a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3, as shown in the figure. A ray of light '*e*' enters the rectangular block at normal incidence. Depending upon the relationships between μ_1, μ_2 and μ_3, it takes one of the four possible paths '*ef*', '*eg*', '*eh*' or '*ei*'.



Match the paths in List I with conditions of refractive indices in List II and select the correct answer using the codes given below the lists:

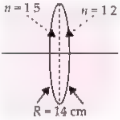
	List I		List II
P.	$e \to f$	1.	$\mu_1 > \sqrt{2}\mu_2$
Q.	$e \to g$	2.	$\mu_2 > \mu_1$ and $\mu_2 > \mu_3$
R.	$e \to h$	3.	$\mu_1 = \mu_2$
S.	$e \to i$	4.	$\mu_2 < \mu_1 < \sqrt{2}\mu_2$ and $\mu_2 > \mu_3$

Codes :

	P	Q	R	S
(a)	2	3	1	4
(b)	1	2	4	3
(c)	4	1	2	3
(d)	2	3	4	1

(2013)

40. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index *n* of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature *R* = 14 cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be



(a) –280.0 cm (b) 40.0 cm
(c) 21.5 cm (d) 13.3 cm *(2012)*

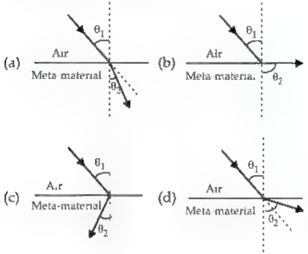
Paragraph for Questions 41 and 42

Most materials have the refractive index, $n > 1$. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that

the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = \left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$, where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are the relative permittivity and permeability of the medium respectively.

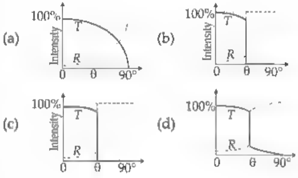
In normal materials, both ε_r and μ_r are positive, implying positive n for the medium. When both ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behaviour, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

41. For light incident from air on a meta-material, the appropriate ray diagram is



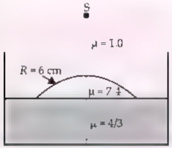
42. Choose the correct statement.
(a) The speed of light in the meta-material is $v = c|n|$.
(b) The speed of light in the meta-material is $v = \frac{c}{|n|}$.
(c) The speed of light in meta-material is $v = c$.
(d) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}|n|$, where λ_{air} is the wavelength of the light in air.
(2012)

43. A light ray travelling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is



(2011)

44. Water (with refractive index $= \frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature $R = 6$ cm as shown. Consider oil to act as a thin lens. An object S is placed 24 cm above water surface. The location of its image is at x cm above the bottom of the tank. Then x is



(Integer Answer Type, 2011)

WAVE OPTICS

45. In the Young's double slit experiment using a monochromatic light of wavelength λ, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is
(a) $(2n+1)\frac{\lambda}{2}$ (b) $(2n+1)\frac{\lambda}{4}$
(c) $(2n+1)\frac{\lambda}{8}$ (d) $(2n+1)\frac{\lambda}{16}$
(2013)

46. Young's double slit experiment is carried out by using green, red and blue light, one color at a time. The fringe widths recorded are β_G, β_R and β_B, respectively. Then,
(a) $\beta_G > \beta_B > \beta_R$ (b) $\beta_B > \beta_G > \beta_R$
(c) $\beta_R > \beta_B > \beta_G$ (d) $\beta_R > \beta_G > \beta_B$ *(2012)*

DUAL NATURE OF RADIATION AND MATTER

47. The work functions of Silver and Sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for Silver to that of Sodium is

(Integer Answer Type, 2013)

48. A proton is fired from very far away towards a nucleus with charge $Q = 120e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is

(Take the proton mass, $m_p = \left(\frac{5}{3}\right) \times 10^{-27}$ kg;

$\frac{h}{e} = 4.2 \times 10^{-15}$ J s/C;

$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F; 1 fm $= 10^{-15}$ m)

(Integer Answer Type, 2012)

49. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^Z$ (where $1 < A < 10$). The value of Z is

(Integer Answer Type, 2011)

ATOMS

50. The radius of the orbit of an electron in a Hydrogen-like atom is 4.5 a_0, where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck constant and R is Rydberg constant. The possible wavelength(s), when the atom de-excites, is (are)

(a) $\frac{9}{32R}$ (b) $\frac{9}{16R}$ (c) $\frac{9}{5R}$ (d) $\frac{4}{3R}$

(2013)

51. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is

(a) 1215 Å (b) 1640 Å
(c) 2430 Å (d) 4687 Å *(2011)*

NUCLEI

52. A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10^3 disintegrations per second. Given that $ln\ 2 = 0.693$, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is

(Integer Answer Type, 2013)

Paragraph for Questions 53 and 54

The mass of a nucleus ${}_Z^AX$ is less than the sum of the masses of $(A-Z)$ number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of masses m_1 and m_2 only if $(m_1 + m_2) < M$. Also two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M' only if $(m_3 + m_4) > M'$. The masses of some neutral atoms are given in the table below :

${}_1^1H$	1.007825 u	${}_1^2H$	2.014102 u
${}_1^3H$	3.016050 u	${}_2^4He$	4.002603 u
${}_3^6Li$	6.015123 u	${}_3^7Li$	7.016004 u
${}_{30}^{70}Zn$	69.925325 u	${}_{34}^{82}Se$	81.916709 u
${}_{64}^{152}Gd$	151.919803 u	${}_{82}^{206}Pb$	205.974455 u
${}_{83}^{209}Bi$	208.980388 u	${}_{84}^{210}Po$	209.982876 u

(1 u = 932 MeV/c^2)

53. The kinetic energy (in keV) of the alpha particle, when the nucleus ${}_{84}^{210}Po$ at rest undergoes alpha decay, is

(a) 5319 (b) 5422 (c) 5707 (d) 5818

54. The correct statement is

(a) The nucleus ${}_3^6Li$ can emit an alpha particle.
(b) The nucleus ${}_{84}^{210}Po$ can emit a proton.
(c) Deuteron and alpha particle can undergo complete fusion.
(d) The nuclei ${}_{30}^{70}Zn$ and ${}_{34}^{82}Se$ can undergo complete fusion. *(2013)*

55. Match List I of the nuclear processes with List II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists :

	List I		List II
P.	Alpha decay	1.	${}_8^{15}O \rightarrow {}_7^{15}N + \ldots$
Q.	β^+ decay	2.	${}_{92}^{238}U \rightarrow {}_{90}^{234}Th + \ldots$
R.	Fission	3.	${}_{83}^{185}Bi \rightarrow {}_{82}^{184}Pb + \ldots$
S.	Proton emission	4.	${}_{94}^{239}Pu \rightarrow {}_{57}^{140}La + \ldots$

Codes :

	P	Q	R	S
(a)	4	2	1	3
(b)	1	3	2	4
(c)	2	1	4	3
(d)	4	3	2	1

(2013)

Paragraph for Questions 56 and 57

The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicted theoretically that the kinetic energy of the electron should be constant. But experimentally, it was observed that the electron kinetic energy has a continuous spectrum. Considering a three-body decay process, *i.e.*, $n \to p + e^- + \bar{\upsilon}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino ($\bar{\upsilon}_e$) to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy.

56. If the anti-neutrino had a mass of 3 eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron?

(a) $0 \le K \le 0.8 \times 10^6$ eV
(b) $3.0\text{ eV} \le K \le 0.8 \times 10^6$ eV
(c) $3.0\text{ eV} \le K < 0.8 \times 10^6$ eV
(d) $0 \le K < 0.8 \times 10^6$ eV

57. What is the maximum energy of the anti-neutrino?

(a) Zero
(b) Much less than 0.8×10^6 eV.
(c) Nearly 0.8×10^6 eV.
(d) Much larger than 0.8×10^6 eV. *(2012)*

58. The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass (in mg) of the radioactive sample is

(Integer Answer Type, 2011)

PRACTICAL PHYSICS

59. The diameter of a cylinder is measured using a Vernier callipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The 24th division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is

(a) 5.112 cm (b) 5.124 cm
(c) 5.136 cm (d) 5.148 cm *(2013)*

60. In the determination of Young's modulus $\left(Y = \dfrac{4MLg}{\pi l d^2}\right)$ by using Searle's method, a wire of length $L = 2$ m and diameter $d = 0.5$ mm is used. For a load $M = 2.5$ kg, an extension $l = 0.25$ mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement

(a) due to the errors in the measurements of d and l are the same.
(b) due to the error in the measurement of d is twice that due to the error in the measurement of l.
(c) due to the error in the measurement of l is twice that due to the error in the measurement of d.
(d) due to the error in the measurement of d is four times that due to the error in the measurement of l. *(2012)*

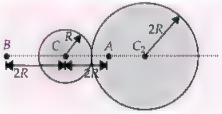
61. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

(a) 0.9% (b) 2.4% (c) 3.1% (d) 4.2%

(2011)

SOLUTIONS

1. **(b, d):** The situation is as shown in the figure.



For a non-conducting uniformly charged solid sphere

$$E_{inside} = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3} \quad (\text{For } r < R)$$

$$E_{\text{outside}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \text{ (For } r > R)$$

At point A,

If net electric field is zero, then

$$\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{(2R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q_2 R}{(2R)^3}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi R^3\rho_1}{(2R)^2} = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi(2R)^3\rho_2 R}{(2R)^3} \Rightarrow \frac{\rho_1}{\rho_2} = 4$$

At point B,

If net electric field is zero, then

$$\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{(2R)^2} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{(5R)^2} = 0$$

$$\frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi R^3\rho_1}{(2R)^2} + \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi(2R)^3\rho_2}{(5R)^2} = 0$$

$$\frac{\rho_1}{\rho_2} = -\frac{32}{25}$$

2. **(a, c, d) :** Position of all the charges are symmetric about the planes, $x = +\frac{a}{2}$ and $x = -\frac{a}{2}$. So net electric flux crossing the plane $x = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = -\frac{a}{2}$

Similarly, the net electric flux crossing the plane $y = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $y = -\frac{a}{2}$.

According to Gauss's law,

The net electric flux crossing the entire region is

$$\phi = \frac{q_{\text{inside}}}{\varepsilon_0} = \frac{3q - q - q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

Charges are symmetrically placed about $z = +\frac{a}{2}$ and $x = +\frac{a}{2}$ planes. So net electric flux crossing the plane $z = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = +\frac{a}{2}$.

3. **(6) :** Electric field at point P due to long uniformly charged solid cylinder is

$$E_1 = \frac{\rho R^2}{2\varepsilon_0(2R)} = \frac{\rho R}{4\varepsilon_0}$$

Electric field at point P due to spherical cavity is

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{\rho\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{(2R)^2} = \frac{\rho R}{96\varepsilon_0}$$

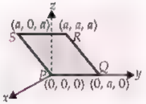
The electric field at the point $P = E_1 - E_2$

$$= \frac{\rho R}{4\varepsilon_0} - \frac{\rho R}{96\varepsilon_0} = \frac{\rho R}{4\varepsilon_0}\left[1 - \frac{1}{24}\right]$$

$$= \frac{23\rho R}{96\varepsilon_0} = \frac{23\rho R}{(16)6\varepsilon_0} = \frac{23\rho R}{16k\varepsilon_0}$$

$\therefore \quad k = 6$

4. **(c) :**



Here, $\vec{E} = E_0\hat{x}$

In this problem, $\hat{x}, \hat{y}$ and $\hat{z}$ are the unit vectors along x, y and z axes respectively

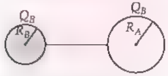
$\therefore$ Shaded area, $\vec{A} = \overrightarrow{PQ} \times \overrightarrow{PS}$

$$\vec{A} = (0\hat{x} + a\hat{y} + 0\hat{z}) \times (a\hat{x} + 0\hat{y} + a\hat{z}) = (a^2\hat{x} - a^2\hat{z})$$

$$\phi = \vec{E}\cdot\vec{A} = (E_0\hat{x})\cdot(a^2\hat{x} - a^2\hat{z}) = E_0 a^2$$

5. **(a,b,c,d) :**

Before connection



After connected by a wire

Electric field is zero inside a conductor

$\therefore \quad E_A^{\text{inside}} = 0$

Hence, (a) is correct.

As equal charge is given to both the spheres the potential $V\left(\propto \frac{Q}{R}\right)$ of smaller sphere will be higher and hence charge will flow from smaller sphere to large sphere when they are connected by a thin wire.

After connection,

$$V_A = V_B$$

$$\frac{1}{4\pi\varepsilon_0}\frac{Q_A}{R_A} = \frac{1}{4\pi\varepsilon_0}\frac{Q_B}{R_B}$$

$$\frac{Q_A}{Q_B} = \frac{R_A}{R_B} \quad \text{...(i)}$$

$\because \; R_B < R_A$ (Given) $\Rightarrow \; Q_A > Q_B$

Hence, (b) is correct.

Charge density, $\sigma = \frac{\text{Charge}}{\text{Area}}$

$\therefore \sigma_A = \frac{Q_A}{4\pi R_A^2}$ and $\sigma_B = \frac{Q_B}{4\pi R_B^2}$

$$\therefore \frac{\sigma_A}{\sigma_B} = \frac{Q_A R_B^2}{Q_B R_A^2}$$

$$= \left(\frac{R_A}{R_B}\right)\left(\frac{R_B}{R_A}\right)^2 = \frac{R_B}{R_A} \quad \text{(Using (i))} \quad ...(ii)$$

Hence, (c) is correct.

Electric field on the surface, $E = \frac{\sigma}{\varepsilon_0}$

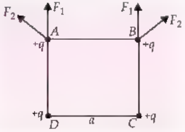
$\therefore E_A^{\text{on surface}} = \frac{\sigma_A}{\varepsilon_0}$ and $E_B^{\text{on surface}} = \frac{\sigma_B}{\varepsilon_0}$

$$\frac{E_A^{\text{on surface}}}{E_B^{\text{on surface}}} = \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A} \quad \text{(Using (ii))}$$

$\because R_B < R_A$ (Given) $\Rightarrow E_A^{\text{on surface}} < E_B^{\text{on surface}}$

Hence, (d) is correct

6. **(3) :**



$$F_1 = \frac{q^2}{4\pi\varepsilon_0 a^2}$$

$$F_2 = \frac{q^2}{4\pi\varepsilon_0(a\sqrt{2})^2} = \frac{q^2}{4\pi\varepsilon_0 2a^2}$$

$\Rightarrow$ Total force on $AB = 2F_1 + \frac{2F_2}{\sqrt{2}}$

$$= \frac{2q^2}{4\pi\varepsilon_0 a^2} + \frac{2q^2}{4\pi\varepsilon_0 2a^2\sqrt{2}} = \frac{q^2}{4\pi\varepsilon_0 a^2}\left[2 + \frac{1}{\sqrt{2}}\right]$$

In equilibrium,

$$\frac{q^2}{4\pi\varepsilon_0 a^2}\left[2 + \frac{1}{\sqrt{2}}\right] = 2\gamma a$$

$$\Rightarrow a^3 = \frac{q^2}{8\pi\varepsilon_0\gamma}\left[2 + \frac{1}{\sqrt{2}}\right]$$

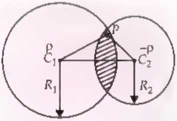
$$\Rightarrow a = \left[\frac{1}{8\pi\varepsilon_0}\left(2 + \frac{1}{\sqrt{2}}\right)\right]^{1/3}\left(\frac{q^2}{\gamma}\right)^{1/3}$$

$$a = k\left[\frac{q^2}{\gamma}\right]^{1/N} \quad \therefore N = 3$$

7. **(c, d)**

8. **(b, d):** When the switch S_1 is pressed and released the charge on the upper plate of C_1 is $2CV_0$. When the switch S_2 is pressed and released the charge on the upper plate of C_1 is CV_0 and charge on the upper plate of C_2 is CV_0. When the switch S_3 is pressed the charge on the upper plate of C_1 is CV_0 and charge on the upper plate of C_2 is $-CV_0$.

9. **(c, d) :**

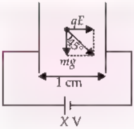


Electrostatic field at P is

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon_0}\overrightarrow{C_1P} + \frac{(-\rho)}{3\varepsilon_0}\overrightarrow{C_2P}$$

$$= \frac{\rho}{3\varepsilon_0}(\overrightarrow{C_1P} - \overrightarrow{C_2P}) = \frac{\rho}{3\varepsilon_0}(\overrightarrow{C_1P} + \overrightarrow{PC_2}) = \frac{\rho}{3\varepsilon_0}(\overrightarrow{C_1C_2})$$

10. **(c) :**



As the proton moves at 45° to the vertical,

$\therefore qE = mg$ or $E = \frac{mg}{q}$

$\because E = \frac{X}{d} \quad \therefore \frac{X}{d} = \frac{mg}{q}$ or $X = \frac{mgd}{q}$

Here, $m = 1.67 \times 10^{-27}$ kg

$g = 10 \text{ m s}^{-2}$, $d = 1 \text{ cm} = 1 \times 10^{-2}$ m

$q = 1.6 \times 10^{-19}$ C

Substituting the given values, we get

$$X = \frac{1.67 \times 10^{-27} \times 10 \times 1 \times 10^{-2}}{1.6 \times 10^{-19}} \text{ V}$$

$$X = \frac{1.67}{1.6} \times 10^{-9} \text{ V} = 1 \times 10^{-9} \text{ V}$$

11. **(d):** Electric field due to a uniformly charged thin spherical shell

Inside the shell

$E_{\text{inside}} = 0 \quad$ [For $r < R$]

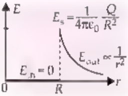
On the surface of the shell

$E_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$ [For $r = R$]

Outside the shell

$E_{outside} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ [For $r > R$]

The variation of E with distance r from the centre as shown in the adjacent figure.



Electric potential due to a uniformly charged thin spherical shell

Inside the shell

$V_{inside} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ [For $r < R$]

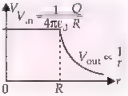
On the surface of the shell

$V_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ [For $r = R$]

Outside the shell

$V_{outside} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

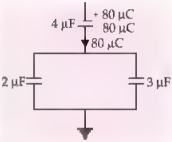
The variation of V with distance r from the centre is as shown in the adjacent figure.



12. (c) : In figure 2 μF and 3 μF are in parallel.

∴ Equivalent capacitance is

$C_{eq} = 2 + 3 = 5\ \mu F$

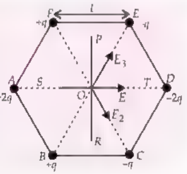


The charge on the upper plate of 3 μF capacitor is

$q = \frac{3\ \mu F}{C_{eq}} \times 80$

$= \frac{3}{5} \times 80 = 3 \times 16 = 48\ \mu C$

13. (a,b,c) :



The electric field along OE and OC are same *i.e.*, $E_2 = E_3 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2q}{L^2}$

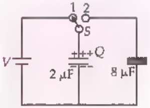
and $E_1 = \frac{1}{4\pi\varepsilon_0}\frac{4q}{L^2}$

$\therefore\ E_{all} = 6 \times \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2} = 6K$ (along OD)

Potential at centre O is zero.

Potential at line PR is zero because this line is equatorial axis for three dipole.

14. (d):



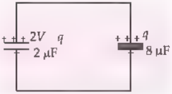
Initially the charge on 2 μF capacitor is

$Q = 2V$

Initially, the energy stored in 2 μF capacitor is

$$U_i = \frac{1}{2} \times 2 \times V^2 = V^2 \quad \text{...(i)}$$

After shifting the switch S to position 2, the charge flows and both capacitors acqure a common potential. Let V_{common} be a common potential



$\therefore\ V_{common} = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{2V}{2+8} = \frac{V}{5}$

Finally, the energy stored in both the capacitors is

$$U_f = \frac{1}{2} \times (2+8)\left(\frac{V}{5}\right)^2 = \frac{V^2}{5} \quad \text{...(ii)}$$

% Loss of energy $= \frac{U_i - U_f}{U_i} \times 100$

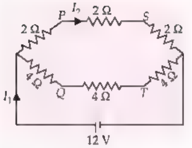
$= \frac{V^2 - \frac{V^2}{5}}{V^2} \times 100 = \frac{4}{5} \times 100 = 80\%$

(Using (i) and(ii))

15. (a, b, c, d) : P and Q are at the same potential. Hence no current flows in arm PQ, so resistance of arm PQ becomes ineffective.

Also, S and T are at the same potential. Hence, no current flows in arm ST, so resistance of arm ST becomes ineffective.

The equivalent circuit is as shown in the figure.

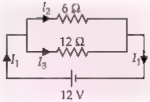


The resistance of the upper arm

$R_1 = 2\ \Omega + 2\ \Omega + 2\ \Omega = 6\ \Omega$

The resistance of the lower arm

$R_2 = 4\ \Omega + 4\ \Omega + 4\ \Omega = 12\ \Omega$

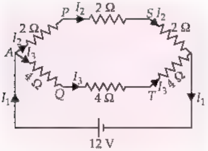


Equivalent resistance of the circuit,

$$R_{eq} = \frac{R_1R_2}{R_1+R_2} = \frac{(6\ \Omega)(12\ \Omega)}{6\ \Omega + 12\ \Omega} = 4\ \Omega$$

$$\therefore\quad I_1 = \frac{12\ \text{V}}{4\ \Omega} = 3\ \text{A}$$

$$I_2 = \left(\frac{12}{6+12}\right)\times 3 = 2\ \text{A}\ ,\ I_3 = I_1 - I_2 = 1\ \text{A}$$



Potential difference across A and P,

$V_A - V_P = I_2 \times 2\ \Omega = (2\ \text{A})(2\ \Omega)$

$12\ \text{V} - V_P = 4\ \text{V}$ or $V_P = 8\ \text{V}$

Potential difference across A and Q,

$V_A - V_Q = I_3 \times 4\ \Omega = (1\ \text{A})(4\ \Omega)$

$12\ \text{V} - V_Q = 4\ \text{V}$ or $V_Q = 12\ \text{V} - 4\ \text{V} = 8\ \text{V}$ ...(i)

Potential difference across P and S,

$V_P - V_S = (2\ \text{A})\ (2\ \Omega) = 4\ \text{V}$

$8\ \text{V} - V_S = 4\ \text{V}$ or $V_S = 4\ \text{V}$ ...(ii)

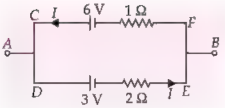
From (i) and (ii), we get, $V_S < V_Q$

16. (b): Applying the condition of balanced Wheatstone bridge, we get

$$\frac{X}{10\ \Omega} = \frac{(52+1)\ \text{cm}}{(48+2)\ \text{cm}} = \frac{53}{50}$$

$$X = 10\ \Omega \times \frac{53}{50} = 10.6\ \Omega$$

17. (5):



Applying Kirchhoff's second law for closed loop $CDEFC$ we get

$-3 - 2I - I + 6 = 0$

$$I = \frac{6-3}{3} = 1\ \text{A}$$

For the lower path

$V_A - 3 - 2 \times 1 = V_B$

$\therefore\quad V_A - V_B = 5\ \text{V}$

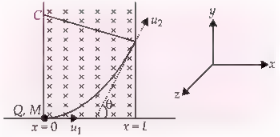
We can also find the V_{AB} by considering the upper path

For the upper path,

$V_A - 6 + 1 \times 1 = V_B$

$V_A - V_B = 5\ \text{V}$

18. (a, c) : The situation is as shown in the figure.



Here, $\vec{u}_1 = 4\hat{i}\ \text{ms}^{-1}$

$\vec{u}_2 = 2(\sqrt{3}\,\hat{i} + \hat{j})\ \text{ms}^{-1}$

When the charged particle enters in a uniform magnetic field, it travels a circular path.

Component of final velocity of particle is in positive y direction.

Centre of circle is present on positive y-axis.

So, magnetic field is present in negative z-direction.

Angle of deviation

$$\tan\theta = \frac{u_{2y}}{u_{2x}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ = \frac{\pi}{6}$$

$$\omega = \frac{\theta}{t}\quad \therefore\quad t = \frac{\theta}{\omega} = \frac{\theta}{(QB/M)} = \frac{M\theta}{QB}\quad \left(\because\ \omega = \frac{QB}{M}\right)$$

$$\therefore\quad B = \frac{M\theta}{Qt} = \frac{M\pi}{6Q\times 10\times 10^{-3}}\quad (\because\ t = 10\ \text{ms} = 10^{-3}\ \text{s})$$

$$= \frac{100M\pi}{6Q} = \frac{50\pi M}{3Q}$$

19. **(a, d)**

20. **(b)**

21. **(b):** According to Faraday's law

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt} \Rightarrow \vec{E}\oint d\vec{l} = -\frac{d}{dt}(AB)$$

$$E2\pi R = -A\frac{dB}{dt}$$

$$E = -\frac{\pi R^2 B}{2\pi R} \qquad \left(\because \frac{dB}{dt} = \frac{B-0}{1} = B(\text{Given})\right)$$

$$= -\frac{BR}{2} \text{ or } E = \frac{BR}{2}$$

22. **(c, d) :**

Here, $\vec{E} = E_0\hat{j}, \vec{B} = B_0\hat{j}$

If $\theta = 0°$, then due to magnetic force path is circular but due to electric force $qE_0\,(\uparrow)$ q will have accelerated motion along y-axis. So combined path of q will be a helical path with variable pitch. So (a) and (b) are wrong.

If $\theta = 10°$ then due to $v\cos\theta$, path is circular and due to qE_0 and $v\sin\theta$, q has accelerated motion along y-axis so combined path is a helical path with variable pitch. So (c) is correct.

If $\theta = 90°$ then $F_B = 0$ and due to qE_0 motion is accelerated along y-axis. So (d) is correct.

23. **(5) :** Magnetic field at point P due to cylinder

$$B_1 = \frac{\mu_0 J\pi a^2}{2\pi a} = \frac{\mu_0 Ja}{2}$$

Magnetic field at point P due to cavity

$$B_2 = \frac{\mu_0}{2\pi}\frac{J\pi\left(\frac{a}{2}\right)^2}{\left(\frac{3a}{2}\right)} = \frac{\mu_0 Ja}{12}$$

The magnetic field at point P is

$$B = B_1 - B_2 = \frac{\mu_0 Ja}{2} - \frac{\mu_0 Ja}{12}$$

$$= \frac{\mu_0 Ja}{2}\left[1-\frac{1}{6}\right] = \frac{5}{12}\mu_0 aJ = \frac{N}{12}\mu_0 aJ \quad \therefore N = 5$$

24. **(b):** Area of the loop $\hat{k}$

$$A = \left[a^2 + 4\times\frac{\pi\left(\frac{a}{2}\right)^2}{2}\right]\hat{k} = \left[a^2 + \frac{\pi a^2}{2}\right]\hat{k}$$

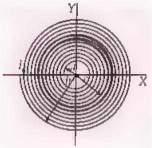
Therefore, the magnetic moment of the current loop is

$$M = I\times A = I\left[a^2 + \frac{\pi a^2}{2}\right]\hat{k} = \left[1+\frac{\pi}{2}\right]Ia^2\hat{k}$$

25. **(d)**

26. **(b, d)**

27. **(a) :**



Take an element of thickness dr at a distance r from the centre of spiral coil.
Number of turns in spiral = N
Number of turns per unit thickness

$$= \frac{N}{b-a}$$

Number of turns in element dr is, $dN = \dfrac{Ndr}{b-a}$

Magnetic field at the centre of the spiral due to current in element dr is

$$dB = \frac{\mu_0}{4\pi}\frac{2\pi dNI}{r} = \frac{\mu_0 I}{2r}dN = \frac{\mu_0 I}{2r}\left(\frac{Ndr}{b-a}\right)$$

$$= \frac{\mu_0 I}{2}\times\frac{N}{(b-a)}\times\frac{dr}{r}$$

Total magnetic field at the centre of the spiral due to current through the wire is

$$B = \int_a^b \frac{\mu_0 INdr}{2(b-a)r} = \frac{\mu_0 IN}{2(b-a)}\int_a^b \frac{dr}{r} = \frac{\mu_0 IN}{2(b-a)}\ln\left(\frac{b}{a}\right)$$

28. **(c) :** Induced electric field and magnetic field form closed loops.

29. **(7) :** Let a current I flows through the circular loop.
The magnetic field at the centre of the square loop due to circular loop

$$B = \frac{\mu_0 IR^2}{2(R^2+(\sqrt{3}R)^2)^{3/2}} = \frac{\mu_0 IR^2}{2(4R^2)^{3/2}} = \frac{\mu_0 I}{16R}$$

Magnetic flux linked with the square loop

$$\phi = NBA\cos 45°$$

$$= 2\times\frac{\mu_0 I}{16R}\times a^2\times\frac{1}{\sqrt{2}} = \frac{\mu_0 Ia^2}{8\sqrt{2}R} = \frac{\mu_0 Ia^2}{2^{7/2}R}$$

So, mutual inductance between the loops is

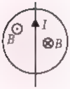
$$M = \frac{\phi}{I} = \frac{\mu_0 Ia^2}{2^{7/2}RI} = \frac{\mu_0 a^2}{2^{7/2}R} = \frac{\mu_0 a^2}{2^{p/2}R}$$

$$\therefore p = 7$$

30. (a, c) :

Total flux associated with the loop is zero

$\therefore$ Induced emf in any case is zero.



31. (6) : According to Ampère's circuital law the magnetic field inside the tube is

$$B = \frac{\mu_0 I}{L} \qquad \text{...(i)}$$

where L is the length of the tube.

Flux linked the wire loop is

$\phi = B\pi r^2$

where r is the radius of the loop

$$\phi = \frac{\mu_0 I}{L}\pi r^2 \qquad \text{(Using (i))}$$

$$= \frac{\mu_0 \pi r^2 I_0 \cos 300t}{L}$$

Induced emf in the loop is

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(\frac{\mu_0}{L}\pi r^2 I_0 \cos 300t\right)$$

$$= \frac{\mu_0 \pi r^2 I_0 300 \sin 300t}{L}$$

Induced current in the loop is

$$i = \frac{\varepsilon}{R} = \frac{300\mu_0 \pi r^2 I_0 \sin 300t}{LR}$$

where R is the resistance of the loop

Magnetic moment of the loop $M = i\pi r^2$

$$= \frac{300\pi^2 r^4 \mu_0 I_0 \sin 300t}{LR}$$

Substituting the given values, we get

$$M = \frac{300 \times 10 \times (0.1)^4}{10 \times 0.005}\mu_0 I_0 \sin 300t \quad (\text{Take } \pi^2 = 10)$$

$$= 6\mu_0 I_0 \sin 300t$$

According to question,

$M = N\mu_0 I_0 \sin 300t \qquad \therefore \quad N = 6$

32. (a) : For a transformer,

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

For a step-up transformer,

Here, $\frac{N_p}{N_s} = \frac{1}{10}$, $V_p = 4000$ V

$$\therefore \quad V_s = V_p\frac{N_s}{N_p} = (4000)\left(\frac{10}{1}\right) = 40{,}000 \text{ V}$$

For a step-down transformer,

Here, $\frac{N_p}{N_s} = ?$, $V_p = 40{,}000$ V, $V_s = 200$ V

$$\therefore \quad \frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{40{,}000 \text{ V}}{200 \text{ V}} = \frac{200}{1}$$

$N_p : N_s = 200 : 1$

33. (b): $P = VI$

$$\therefore \quad I = \frac{P}{V} = \frac{600 \text{ kW}}{4000 \text{ V}} = \frac{600 \times 10^3 \text{ W}}{4000 \text{ V}} = 150 \text{ A}$$

Resistance of the cable,

$R = 0.4\ \Omega\ \text{km}^{-1} \times 20\ \text{km} = 8\ \Omega$

Line power loss $= I^2R = (150\ \text{A})^2(8\ \Omega)$

$= 180 \times 10^3\ \text{W} = 180\ \text{kW}$

$$\%\ \text{loss} = \frac{180\ \text{kW}}{600\ \text{kW}} \times 100 = 30\%$$

34. (a, c) :

Here, $\omega = 100$ rad/s, $L = 0.5$ H,

$C = 100\ \mu$F, $V = 20$ V

$\therefore \quad X_L = \omega L = 100 \times 0.5 = 50\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 100 \times 10^{-6}} = 100\ \Omega$$

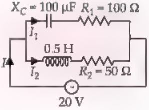
Impedance across capacitor arm,

$$Z_1 = \sqrt{R^2 + X_C^2}$$

$$= \sqrt{(100)^2 + (100)^2}$$

$$Z_1 = 100\sqrt{2}\ \Omega$$

$$\therefore \quad I_1 = \frac{20}{100\sqrt{2}} = \frac{1}{5\sqrt{2}} \text{ A}$$



Voltage across 100 Ω

$$V = I_1 \times 100 = \frac{1}{5\sqrt{2}} \times 100 = 10\sqrt{2} \text{ V}$$

Impedance across inductance arm,

$$Z_2 = \sqrt{R^2 + (X_L)^2} = \sqrt{(50)^2 + (50)^2}$$

$$Z_2 = 50\sqrt{2}\ \Omega$$

$$\therefore \quad I_2 = \frac{20}{50\sqrt{2}} = \frac{2}{5\sqrt{2}} = \frac{\sqrt{2}}{5}$$

Now voltage across 50 Ω $= \frac{\sqrt{2}}{5} \times 50 = 10\sqrt{2}$ V

$$I_1 = \frac{1}{5\sqrt{2}} \text{ A at } 45° \text{ leading}$$

$$I_2 = \frac{\sqrt{2}}{5} \text{ A at } 45° \text{ lagging}$$

$\therefore$ Current through circuit

$$I_{net} = \sqrt{I_1^2 + I_2^2} = \sqrt{\left(\frac{1}{5\sqrt{2}}\right)^2 + \left(\frac{\sqrt{2}}{5}\right)^2} = 0.3$$

35. (b, c) : In case A,

The capacitive reactance is

$$X_C^A = \frac{1}{\omega C}$$

Impedance of the circuit is

$$Z_A=\sqrt{(R)^2+\left(\frac{1}{\omega C}\right)^2} \text{ or } I_R^A=\frac{V}{\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}} \quad \text{...(i)}$$

$$V_C^A=\frac{I_R^A}{\omega C}=\frac{V}{\sqrt{(R\omega C)^2+1}} \quad \text{...(ii)}$$

In case B,

The capacitive reactance is

$$X_C^B=\frac{1}{\omega(4C)}=\frac{1}{4\omega C}$$

Impedance of the circuit is

$$Z_B=\sqrt{R^2+\left(\frac{1}{4\omega C}\right)^2} \text{ or } I_R^B=\frac{V}{\sqrt{R^2+\left(\frac{1}{4\omega C}\right)^2}} \quad \text{...(iii)}$$

$$V_C^B=\frac{V}{\sqrt{(4R\omega C)^2+1}} \quad \text{...(iv)}$$

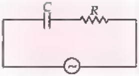
From (i) and (iii), we conclude that

$$I_R^A<I_R^B$$

From (ii) and (iv), we conclude that

$$V_C^A>V_C^B$$

36. (4) : Here, $\omega = 500$ radian/s



The capacitive reactance is

$$X_C=\frac{1}{\omega C}$$

The impedance of the circuit is

$$Z=\sqrt{R^2+(X_C)^2}=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}$$

$$R\sqrt{\frac{5}{4}}=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2} \text{ or } \frac{5}{4}R^2=R^2+\left(\frac{1}{\omega C}\right)^2$$

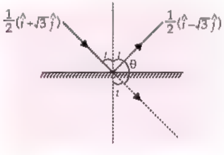
$$\frac{1}{4}R^2=\left(\frac{1}{\omega C}\right)^2 \text{ or } R^2C^2=4\left(\frac{1}{\omega}\right)^2$$

$$\text{or } RC=\frac{2}{\omega}=\frac{2}{500}\text{ s}=0.4\times10^{-2}\text{ s}$$

$$=4\times10^{-3}\text{ s}=4 \text{ millisecond}$$

The time constant of RC circuit, $\tau = RC = 4$ ms

37. (a) :



Let θ be angle between the given rays.

$$\therefore \quad \cos\theta=\frac{\left(\frac{\hat{i}+\sqrt{3}\hat{j}}{2}\right)\cdot\left(\frac{\hat{i}-\sqrt{3}\hat{j}}{2}\right)}{\left|\frac{\hat{i}+\sqrt{3}\hat{j}}{2}\right|\left|\frac{\hat{i}-\sqrt{3}\hat{j}}{2}\right|}=\frac{\frac{1-3}{4}}{1}=-\frac{1}{2}$$

$$\theta=\cos^{-1}\left(-\frac{1}{2}\right)=120^\circ$$

∴ Angle of incidence,

$$i=\frac{180^\circ-\theta}{2}=\frac{180^\circ-120^\circ}{2}=30^\circ$$

38. (c) : Image is formed behind the lens.

∴ $v = +8$ m

As the image is real

$$\therefore \quad m=\frac{I}{O}=\frac{v}{u}=-\frac{1}{3}$$

$$u=-3v=-3(8\text{ m})=-24\text{ m}$$

According to lens formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \therefore \frac{1}{8}-\frac{1}{-24}=\frac{1}{f}$$

$$\frac{1}{8}+\frac{1}{24}=\frac{1}{f} \text{ or } \frac{4}{24}=\frac{1}{f} \text{ or } f=6\text{ m} \quad \text{...(i)}$$

Refractive index of the material of the lens is

$$\mu=\frac{\text{Wavelength of the light in free space}}{\text{Wavelength of light inside the lens}}$$

$$=\frac{\lambda_0}{\frac{2}{3}\lambda_0}=\frac{3}{2} \quad \text{...(ii)}$$

According to lens maker's formula

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

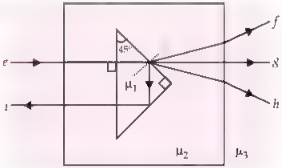
For plano-convex lens, $R_1 = R$ and $R_2 = \infty$

$$\therefore \quad \frac{1}{f}=\frac{(\mu-1)}{R} \text{ or } R=f(\mu-1) \quad \text{...(iii)}$$

Substituting the value of μ and f from Eqs. (i) and (ii) in (iii), we get

$R = (6\text{ m})(1.5 - 1) = 3$ m

39. (d):



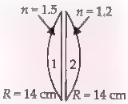
For $e \to f$
$\mu_2 > \mu_1$, as ray bends towards the normal.
$\mu_2 > \mu_3$, as ray bends away from the normal.
$P \to 2$
For $e \to g$
$\mu_1 = \mu_2$ as there is no deviation.
$Q \to 3$
For $e \to h$
$\mu_2 < \mu_1$, as ray bends away from the normal.
$\mu_2 > \mu_3$, as ray bends away from the normal.
Also, $\mu_1 < \sqrt{2}\mu_2$ (No total internal reflection)
$R \to 4$
For $e \to i$
Total internal reflection takes place
$\therefore \quad \sin 45° > \sin C$

But $\sin C = \dfrac{\mu_2}{\mu_1}$

$$\therefore \quad \frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1} \Rightarrow \mu_1 > \sqrt{2}\mu_2$$
$S \to 1$

40. **(b):** According to lens maker's formula

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$



For the first lens
$n = 1.5$, $R_1 = +14$ cm, $R_2 = \infty$

$$\therefore \quad \frac{1}{f_1} = (1.5-1)\left(\frac{1}{14} - \frac{1}{\infty}\right) = \frac{0.5}{14}$$

For the second lens,
$n = 1.2$, $R_1 = \infty$, $R_2 = -14$ cm

$$\therefore \quad \frac{1}{f_2} = (1.2-1)\left(\frac{1}{\infty} - \frac{1}{-14}\right) = \frac{0.2}{14}$$

The focal length of the bi-convex lens is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{0.5}{14} + \frac{0.2}{14} = \frac{0.7}{14} = \frac{1}{20}$$

According to thin lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Here, $u = -40$ cm

$$\therefore \quad \frac{1}{v} - \frac{1}{-40} = \frac{1}{20} \text{ or } \frac{1}{v} = \frac{1}{20} - \frac{1}{40} \text{ or } v = 40 \text{ cm}$$

41. **(c):** For meta-material, the refractive index is negative. Let n_1 is refractive index of air and n_2 is refractive index of meta-material.
$\therefore$ From Snell's law

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

Since n_2 is negative, therefore θ_2 is also negative. Hence, appropriate diagram (c) is correct.

42. **(b):** Refractive index for a medium

$$n = \left(\frac{c}{v}\right)$$

For meta material, $n = |n|$

$$\therefore \quad v = \frac{c}{|n|}$$

43. **(c):** When $\theta < \theta_c$ (critical angle) there will be partial transmission and reflection will takes place. When $\theta > \theta_c$, there will be 100% reflection takes place so transmitted intensity = 0.

44. **(2):** In case of refraction from a curved surface,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Refraction at air-oil interface
Here, $u = -24$ cm, $v = v_1$, $\mu_1 = 1$,

$$\mu_2 = \frac{7}{4}, \; R = +6 \text{ cm}$$

$$\therefore \quad \frac{7}{4v_1} - \frac{1}{-24} = \frac{\left(\frac{7}{4} - 1\right)}{6}$$

$$\frac{7}{4v_1} + \frac{1}{24} = \frac{3}{24} \Rightarrow \frac{7}{4v_1} = \frac{1}{12} \to v_1 = 21 \text{ cm}$$

This image will act as object for the oil-water interface
Refraction at oil-water interface
Here $u = +21$ cm, $v = v_2$, $\mu_1 = \dfrac{7}{4}$, $\mu_2 = \dfrac{4}{3}$, $R = \infty$

$$\therefore \quad \frac{4}{3v_2} - \frac{(7/4)}{21} = 0 \Rightarrow \frac{4}{3v_2} = \frac{7}{84}$$

$v_2 = 16$ cm
Hence, $x = (18 - 16)$ cm $= 2$ cm

45. **(b):** As $I = I_{max}\cos^2\left(\dfrac{\phi}{2}\right)$

Here, $I = \dfrac{I_{max}}{2}$

$$\therefore \quad \frac{I_{max}}{2} = I_{max}\cos^2\left(\frac{\phi}{2}\right) \Rightarrow \frac{1}{2} = \cos^2\frac{\phi}{2}$$

$$\Rightarrow \cos\phi = 0 \Rightarrow \phi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

or $\quad \phi = (2n+1)\dfrac{\pi}{2}$ where $n = 0, 1, 2, \ldots$

Since phase difference $= \dfrac{2\pi}{\lambda} \times$ Path difference

$\therefore$ Path difference $= \dfrac{\lambda}{2\pi} \times$ Phase difference

$$= \frac{\lambda}{2\pi} \times (2n+1)\frac{\pi}{2} = (2n+1)\frac{\lambda}{4}$$

46. **(d):** Fringe width, $\beta = \frac{\lambda D}{d}$

$\therefore \quad \beta \propto \lambda$

As $\lambda_R > \lambda_G > \lambda_B \quad \therefore \; \beta_R > \beta_G > \beta_B$

47. **(1) :** According to Einstein's photoelectric equation,

$K_{max} = h\upsilon - \phi_0$

where the symbols have their usual meaning.

But $K_{max} = eV_s$ where V_s is the stopping potential

$V_s = \frac{h}{e}\upsilon - \frac{\phi_0}{e}$

Thus, the graph between $V_s - \upsilon$ is a straight line.

Compare the above relation with $y = mx + C$

$\therefore \quad$ Slope of $V_s - \upsilon$ graph $= \frac{h}{e}$

It is same for both the metals.

$\therefore \quad$ Ratio of the slopes = 1.

48. **(7) :** As $\frac{1}{4\pi\varepsilon_0}\frac{(120e)(e)}{10\times10^{-15}} = \frac{p^2}{2m_p}$ ...(i)

where p is the momentum of the proton and m_p is the mass of the proton

de Broglie wavelength of proton,

$\lambda = \frac{h}{p} \quad$ or $\quad p = \frac{h}{\lambda}$

Substituting this value of p in equation (i), we get

$$\frac{1}{4\pi\varepsilon_0}\frac{120e^2}{10\times10^{-15}} = \frac{h^2}{2\lambda^2 m_p}$$

$$\lambda^2 = \frac{4\pi\varepsilon_0 \times 10 \times 10^{-15} \times h^2}{2m_p \times 120e^2}$$

Substituting the given numerical values, we get

$$\lambda^2 = \frac{1\times10\times10^{-15}\times4.2\times10^{-15}\times4.2\times10^{-15}}{9\times10^9\times2\times\frac{5}{3}\times10^{-27}\times120}$$

$$= \frac{4.2\times4.2\times10^{-30}\times10^{-14}}{36\times10^{-16}}$$

$\lambda = 7\times10^{-15}$ m = 7 fm

49. **(7) :** Here, radius of sphere R = 1 cm = 1×10^{-2} m

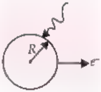
Work function, W = 4.7 eV

Energy of incident radiation

$= \frac{hc}{\lambda} = \frac{1240 \text{ eV nm}}{200 \text{ nm}}$ (Take hc = 1240 eV nm)

= 6.2 eV

According to Einstein's photoelectric equation



$\frac{hc}{\lambda} = W + eV_s$

6.2 eV = 4.7 eV + eV_s

V_s = 1.5 V

The sphere will stop emitting photoelectrons, when the potential on its surface becomes equal to 1.5 V.

$\therefore \quad \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = 1.5$

$\frac{1}{4\pi\varepsilon_0}\frac{Ne}{R} = 1.5$

where N = Number of photoelectrons emitted

e = charge of each electron

$$N = \frac{1.5\times R}{\frac{1}{4\pi\varepsilon_0}\times e} = \frac{1.5\times1\times10^{-2}}{9\times10^9\times1.6\times10^{-19}}$$

$$N = \frac{15}{16}\times\frac{1}{9}\times10^8 = \frac{5}{48}\times10^8$$

$$N = \frac{50}{48}\times10^7 = 1.04\times10^7 \quad \therefore \quad Z = 7$$

50. **(a, c) :** According to Bohr's quantisation condition

$$L = \frac{nh}{2\pi}$$

Given : $L = \frac{3h}{2\pi} \quad \therefore \quad \frac{3h}{2\pi} = \frac{nh}{2\pi} \to n = 3$

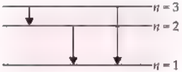
Radius of n^{th} orbit for hydrogen-like atom is

$r_n = \frac{n^2}{Z}a_0$ where a_0 is the Bohr radius

Here, $r_n = 4.5a_0$

$\therefore \quad 4.5a_0 = \frac{(3)^2}{Z}a_0 \Rightarrow Z = \frac{9}{4.5} = 2$

The possible transitions are as shown in the figure.



According to Rydberg formula

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

The corresponding emitted wavelengths are

For $n = 3 \to n = 2$

$$\frac{1}{\lambda_{3\to2}} = R(2)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= 4R\left(\frac{1}{4} - \frac{1}{9}\right) = 4R\left(\frac{9-4}{36}\right) = \frac{5R}{9}$$

$\Rightarrow \quad \lambda_{3\to2} = \frac{9}{5R}$

For $n = 2 \to n = 1$

$$\frac{1}{\lambda_{2\to1}} = R(2)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 4R\left(\frac{1}{1} - \frac{1}{4}\right)$$

Contd. on page no. 88

PRACTICE PAPER 2014

AIIMS

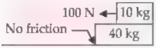
All India Institute of Medical Sciences

Exam on 1st June 2014

1. From the top of a tower, a particle is thrown vertically downwards with a velocity of 10 m s^{-1}. The ratio of the distances covered by it in the 3rd and 2nd seconds of its motion is (Take $g = 10$ m s^{-2})
(a) 7 : 5 (b) 3 : 4 (c) 4 : 3 (d) 6 : 5

2. The area enclosed by a circle of diameter 1.06 m to correct number of significant figures is
(a) 0.88 m^2 (b) 0.088 m^2
(c) 0.882 m^2 (d) 0.530 m^2

3. A 40 kg slab rests on a frictionless floor. A 10 kg block rests on top of the slab. The static coefficient of friction between the block and the slab is 0.60 while the kinetic coefficient of friction is 0.40. The 10 kg block is acted upon by a horizontal force of 100 N. If $g = 9.8$ m s^{-2}, the resulting acceleration of the slab will be



(a) 1.47 m s^{-2} (b) 1.69 m s^{-2}
(c) 9.8 m s^{-2} (d) 0.98 m s^{-2}

4. An infinite non-conducting sheet has a surface charge density $\sigma = 0.1$ μC m^{-2} on one side. How far apart are equipotential surfaces whose potentials differ by 50 V?
(a) 8.86 mm (b) 8.8 μm
(c) 8.8 cm (d) 8.8 pm

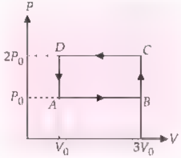
5. Two wires of same metal have the same length but their cross sections are in the ratio 3 : 1. They are joined in series. The resistance of the thicker wire is 10 Ω. The total resistance of the combination is
(a) $\frac{5}{2}$ Ω (b) $\frac{40}{3}$ Ω (c) 40 Ω (d) 100 Ω

6. The torque required to hold a small circular coil of 10 turns, 2×10^{-4} m^2 area and carrying 0.5 A current in the middle of a long solenoid of 10^3 turns per m carrying 3 A current, with its axis perpendicular to the axis of the solenoid, is
(a) $12\pi \times 10^{-7}$ N m (b) $6\pi \times 10^{-7}$ N m
(c) $4\pi \times 10^{-7}$ N m (d) $2\pi \times 10^{-7}$ N m

7. The length of a second pendulum is 1 m on the Earth. If the mass and diameter of a planet is double than that of the Earth, then the length of second pendulum on the planet becomes
(a) 1 m (b) 2 m (c) 0.5 m (d) 4 m

8. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (surface tension of soap solution is 0.03 N m^{-1})
(a) 0.4π mJ (b) 4π mJ
(c) 0.2π mJ (d) 2π mJ

9. An ideal gas undergoes cyclic process *ABCDA* as shown in given *P-V* diagram.

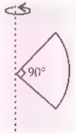


The amount of work done by the gas is
(a) $6P_0V_0$ (b) $-2P_0V_0$
(c) $2P_0V_0$ (d) $4P_0V_0$

10. A stretched string of length 1 m and mass 5×10^{-4} kg, fixed at both ends, is under a tension of 20 N. If it is plucked at points situated at 25 cm from one end, it would vibrate with a frequency
(a) 400 Hz (b) 200 Hz
(c) 100 Hz (d) 256 Hz

11. A body of mass 5 kg is moving with a momentum of 10 kg m s^{-1}. A force of 0.2 N acts on it in the direction of motion of body for 10 s. The increase in its kinetic energy is
(a) 2.8 J (b) 3.2 J
(c) 3.8 J (d) 4.4 J

12. One quarter sector is cut from a uniform circular disc of mass M and radius R, as shown in figure. It is made to rotate about a line perpendicular to its plane and passing through the center of the original disc. Its moment of inertia about the axis of rotation is



(a) $\frac{1}{2}MR^2$ (b) $\frac{1}{4}MR^2$
(c) $\frac{1}{8}MR^2$ (d) $\sqrt{2}MR^2$

13. A ball is projected upwards from the top of tower with a velocity 50 m s^{-1} making an angle 30° with the horizontal. The height of tower is 70 m. After how many seconds the ball reach the ground?
(a) 5 s (b) 12 s
(c) 7 s (d) 15 s

14. The bob of a simple pendulum is of mass 10 g. It is suspended with a thread of 1 m. If we hold the bob so as to stretch the string horizontally and release it, what will be the tension at the lowest position ? (Take, g = 10 m s^{-2})
(a) zero (b) 0.1 N
(c) 0.3 N (d) 1.0 N

15. A ray of light moves from denser to rarer medium. Which of the following is correct?
(a) Energy increases
(b) Frequency increases
(c) Phase change by 90°
(d) Velocity increases

16. A beam of light of wavelength 600 nm from a distant source falls on a single slit 1 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. The distance between the first dark fringes on either side of the central bright fringe is
(a) 1.2 cm (b) 1.2 mm
(c) 2.4 cm (d) 2.4 mm

17. In a plane electromagnetic wave electric field varies with time having an amplitude 1 V m^{-1}. The frequency of wave is 0.5×10^{15} Hz. The wave propagation is along x-axis. What is the average energy density of magnetic field?
(a) 1.1×10^{-12} J m^{-3} (b) 2.2×10^{-12} J m^{-3}
(c) 3.3×10^{-12} J m^{-3} (d) 4.4×10^{-12} J m^{-3}

18. If a star convert all the He nuclei completely into oxygen nuclei, the energy released per oxygen nucleus is [Mass of He nucleus is 4.0026 and mass of Oxygen nucleus is 15.9994 amu]
(a) 7.6 MeV (b) 56.12 MeV
(c) 10.24 MeV (d) 23.9 MeV

19. If a Zener diode (V_Z = 5 V and I_Z = 10 mA) is connected in series with a resistance and 20 V is applied across the combination, then the maximum resistance one can use without spoiling Zener action is
(a) 20 kΩ (b) 15 kΩ (c) 10 kΩ (d) 1.5 kΩ

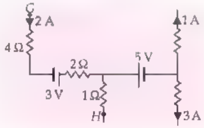
20. If the rms current in a 50 Hz ac circuit is 5 A, the value of the current (1/300) seconds after its value becomes zero is
(a) $5\sqrt{2}$ A (b) $5\sqrt{3/2}$ A
(c) 5/6 A (d) $5/\sqrt{2}$ A

21. A 150 ohm resistor and an inductance L are connected in series to a 200 V, 50 Hz source of negligible impedance. The current comes to 1.0 A. When the source is changed to 400 V, 100 Hz, the current will be
(a) less than 1.0 A
(b) 1.0 A
(c) between 1 A and 2 A
(d) between 4 A and 2 A

22. A bar magnet of length 3 cm has points A and B along its axis at distances of 24 cm and 48 cm on the opposite sides as shown in figure

Ratio of magnetic fields at these points will be
(a) 8 (b) 1/2 (c) 3 (d) 4

23. In the part of a circuit shown in figure, the potential difference between points G and H will be



(a) 0 V (b) 12 V
(c) 7 V (d) 3 V

24. Light of frequency υ falls on material of threshold frequency υ_0. Maximum kinetic energy of emitted electron is proportional to

(a) $\upsilon - \upsilon_0$ (b) υ

(c) $\sqrt{\upsilon - \upsilon_0}$ (d) υ_0

25. A 12 Ω resistor and a 0.21 H inductor are connected in series to an ac source operating at 20 V, 50 cycle. The phase angle between the current and the source voltage is

(a) 30° (b) 40° (c) 80° (d) 90°

26. If the frequency of the source is doubled in Young's double slit experiment, then initial fringe width (β) will become

(a) unchanged (b) $\frac{\beta}{2}$

(c) 2β (d) 3β

27. A block of mass 2 kg is placed on the floor. The coefficient of static friction is 0.4. If a force of 2.8 N is applied on the block parallel to floor, the force of friction between the block and floor is (Take, $g = 10$ m s^{-2}) is

(a) 2.8 N (b) 8 N

(c) 2 N (d) zero

28. A player caught a cricket ball of mass 150 g moving at a rate of 20 m s^{-1}. If the catching process in completed in 0.1 s, the force of the blow exerted by the ball on the hand of the player is equal to

(a) 150 N (b) 3 N

(c) 30 N (d) 300 N

29. Two discs of the same material and thickness have radii 0.2 m and 0.6 m. Their moments of inertia about the axes will be in the ratio

(a) 1 : 81 (b) 1 : 27

(c) 1 : 9 (d) 1 : 3

30. A stone is dropped into a lake from a tower 500 m high. The sound of the splash will be heard at the top of the tower approximately after a time of (take velocity of sound in air = 330 m s^{-1})

(a) 11.5 s (b) 1.5 s

(c) 10 s (d) 14 s

31. An electron is moving with an initial velocity $\vec{v} = v_0 i$ and is in a magnetic field $\vec{B} = B_0 j$. Then its de Broglie wavelength

(a) remains constant

(b) increases with time

(c) decreases with time

(d) increases and decreases periodically

32. The half-life of a radioactive substance is 40 years. How long will it take to reduce to one-fourth of its original amount and what is the value of decay constant respectively?

(a) 90 years, 0.917 per year

(b) 40 years, 0.9173 per year

(c) 80 years, 0.0173 per year

(d) none of these

33. Application of a forward bias to a *p-n* junction

(a) increases the number of donors on the *n*-side

(b) increases the electric field in the depletion zone

(c) increases the potential difference across the depletion zone

(d) widens the depletion zone.

34. A prism is filled with a liquid of refractive index of $\sqrt{2}$. If angle of prism is 60°, the angle of minimum deviation is

(a) 75° (b) 60° (c) 45° (d) 30°

35. In the relation, $y = r\sin(\omega t + kx)$, the dimensional formula for kx or ωt is same as

(a) r/ω (b) r/y (c) $\omega t/r$ (d) $yr/\omega t$

36. The velocity of a particle at an instant is 10 m s^{-1}. After 3 s its velocity will become 16 m s^{-1}. The velocity at 2 s, before the given instant will be

(a) 6 m s^{-1} (b) 4 m s^{-1} (c) 2 m s^{-1} (d) 1 m s^{-1}

37. A circular disc of mass M and radius R is rotating with an angular velocity ω about an axis passing through its centre and perpendicular to the plane of the disc. A small point like part of mass m detaches from the rim of the disc and continues to move with same angular speed. The angular velocity of remaining disc just after detaching will become

(a) $\left(\frac{M-2m}{M+m}\right)\omega$ (b) $\left(\frac{M+2m}{M+m}\right)\omega$

(c) $\left(\frac{M-2m}{M-m}\right)\omega$ (d) $\left(\frac{M+2m}{M-m}\right)\omega$

38. The point charges Q and $-2Q$ are placed some distance apart. If the electric field at the location of Q is E, then the electrical field at the location of $-2Q$ will be

(a) $-E/2$ (b) $-3E/2$ (c) $-E$ (d) $-2E$

39. A cell of emf 1.5 V and internal resistance 2 Ω is connected to two resistors of 5 Ω and 8 Ω in series. The potential difference across the 5 Ω resistor will be

(a) 3.3 V (b) 1 V

(c) 0.5 V (d) 0.33 V

40. An electron is projected with uniform velocity along the axis of a current carrying long solenoid.

Which of the following statement about this is true?

(a) The electron will be accelerated along the axis.

(b) The electron's path will be circular about the axis.

(c) The electron will experience a force at 45° to the axis and hence execute a helical path.

(d) The electron will continue to move with uniform velocity along the axis of the solenoid.

Directions : In the following questions (41-60), a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as

(a) If both assertion and reason are true and reason is the correct explanation of assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of assertion.

(c) If assertion is true but reason is false

(d) If both assertion and reason are false.

41. Assertion : Temperature of real gases often changes when undergoing a free expansion under adiabatic condition whereas that of an ideal gas does not.
Reason : First law of thermodynamics holds for ideal gas only.

42. Assertion : Frictional forces are conservative forces
Reason : Potential energy can be associated with frictional forces

43. Assertion : The combination of $y = \sin\omega t + \cos 2\omega t$ is not a simple harmonic function even thought it is periodic.
Reason : All periodic functions satisfy the relation $\frac{d^2y}{dt^2} = -ky$.

44 Assertion : A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.
Reason : For every action there is an equal and opposite reaction.

45. Assertion : An electron moves from a region of lower potential to a region of higher potential.
Reason : An electron has a negative charge.

46. Assertion : A 60 W - 220 V bulb glows more than a 100 W - 220 V bulb when they are connected in series across a potential difference.
Reason : When they are connected in series, resistance of 100 W bulb will be more.

47. Assertion : An emf ε is induced in a closed loop where magnetic flux is varied. The induced electric field $\vec{E}$ is not a conservative field.
Reason : The line integral of $\vec{E}\cdot\vec{dl}$ around the closed loop is non-zero.

48. Assertion : If an electron and proton possessing same kinetic energy enter an electric field in a perpendicular direction, the path of the electron is more curved than that of the proton.
Reason : Electron forms a larger curve due to its small mass.

49. Assertion : The centre of mass of an electron and proton, when released moves faster towards proton.
Reason : This is because proton is heavier.

50. Assertion : Generally the path of a projectile from the Earth is parabolic but it is elliptical for projectile going to a very great height.
Reason : Upto ordinary height the projectile moves under a uniform gravitational force, but for great heights, projectile moves under a variable force.

51. Assertion : Improper banking of roads causes wear and tear of tyres.
Reason : The necessary centripetal force in that event is provided by the force of friction between the tyres and the road.

52. Assertion : The relative velocity between any two bodies may be equal to sum of the velocities of two bodies.
Reason : Sometimes, relative velocity between two bodies may be equal to difference in velocities of the two.

53. Assertion : A double convex lens (μ = 1.5) has focal length 10 cm. When the lens is immersed in water (μ = 4/3), its focal length becomes 40 cm.

Reason : $\frac{1}{f} = \left(\frac{\mu_g - \mu_a}{\mu_a}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

54. Assertion : Work function of aluminium is 4.2 eV. Emission of electrons will not be possible if two photons, each of energy 2.5 eV, strike an electron of aluminium.
Reason : For photoelectric emission, the energy of each photon should be greater than the work function of aluminium.

55. Assertion : ^{90}Sr from radioactive fallout from a nuclear bomb ends up in the bones of human beings through the milk consumed by them. It causes impairment of the production of red blood cells.

Reason : The energetic β-particles emitted in the decay of ^{90}Sr damage the bone marrow.

56. **Assertion :** In a common-base configuration, the current gain of transistor is less than unity.
Reason : The collector terminal is reverse biased for amplification.

57. **Assertion :** When two soap bubble's of different radii are brought into contact, the common interface of contact bulges into the bubble of larger radii.
Reason : Pressure inside a soap bubble of lesser radius is more than pressure inside a soap bubble of larger radius.

58. **Assertion :** For a system of particles under central force field, the total angular momentum is conserved.
Reason : The torque acting on such a system is zero

59. **Assertion :** If a charged particle is released from rest in a region of uniform electric and magnetic fields parallel to each other, it will move in a straight line
Reason : The electric field exerts no force on the particle but the magnetic field does.

60. **Assertion :** A rocket moves forward by pushing the surrounding air backwards.
Reason : It derives the necessary thrust to move forward according to Newton's third law of motion.

SOLUTIONS

1. **(a) :** The distance covered by the particle in the 3rd s,

$$S_3 = 10 + \frac{10}{2}(2\times 3 - 1) = 35 \text{ m}$$

Distance covered in 2nd s,

$$S_2 = 10 + \frac{10}{2}(2\times 2 - 1) = 25 \text{ m}$$

$$\therefore \frac{S_3}{S_2} = \frac{35 \text{ m}}{25 \text{ m}} = \frac{7}{5}$$

2. **(c) :** Here, $r = \frac{1.06}{2} = 0.530$ m

Area enclosed $= \pi r^2 = 3.14 \times (0.53)^2$
$= 0.882026 \text{ m}^2$
$= 0.882 \text{ m}^2$ (rounded to three significant figures)

3. **(d):** Force of static friction,
$f_s = \mu\, mg = 0.60 \times 10 \times 9.8 = 58.8$ N
Since the applied force is greater than f_s therefore the block will be in motion.

$\because$ Force of kinetic friction,
$f_k = 0.40 \times 10 \times 9.8 = 4 \times 9.8$ N
This would cause acceleration of 40 kg block

$$\therefore \text{ Acceleration} = \frac{4\times 9.8 \text{ N}}{40 \text{ kg}} = 0.98 \text{ m s}^{-2}$$

4. **(a) :** Here, $\sigma = 0.1\ \mu\text{C m}^{-2} = 0.1 \times 10^{-6} \text{ C m}^{-2}$,
$dV = 50$ V

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{dV}{dr}$$

or $$dr = \frac{2\varepsilon_0 dV}{\sigma} = \frac{2\times 8.86\times 10^{-12}\times 50}{0.1\times 10^{-6}}$$

$= 8.86 \times 10^{-3}$ m $= 8.86$ mm

5. **(c) :** For the same length and same material,

$$\frac{R_2}{R_1} = \frac{A_1}{A_2} = \frac{3}{1} \text{ or } R_2 = 3R_1$$

The resistance of thick wire, $R_1 = 10\ \Omega$
The resistance of thin wire $= 3R_1 = 3 \times 10 = 30\ \Omega$
Total resistance $= 10 + 30 = 40\ \Omega$

6. **(a) :** Magnetic dipole moment of current loop is
$M = NIA = 10 \times 0.5 \times 2 \times 10^{-4} = 10^{-3} \text{ A m}^2$
Magnetic field inside the solenoid carrying current,
$B = \mu_0 nI = 4\pi \times 10^{-7} \times 10^3 \times 3 = 12\pi \times 10^{-4}$ T
$\therefore$ Torque, $\tau = MB\sin\theta$
$10^{-3} \times 12\pi \times 10^{-4} \times \sin 90°$
$= 12\pi \times 10^{-7}$ N m

7. **(c) :** Acceleration due to gravity on the planet,

$$g' = \frac{G(2M)}{(2R)^2} = \frac{1}{2}\left(\frac{GM}{R^2}\right) = \frac{g}{2} \quad \left(\because g = \frac{GM}{R^2} \text{ on Earth}\right)$$

or $$\frac{g'}{g} = \frac{1}{2}$$

As for a second pendulum,

$$l = \frac{g}{\pi^2} \text{ or } \frac{l'}{l} = \frac{g'}{g} = \frac{1}{2}$$

or $$l' = \frac{1}{2} = 0.5 \text{ m}$$

8. **(a) :** Work done in increasing the size of the soap bubble
= surface tension × increase in area of both the free surface of the bubble

i.e. $$W = T\times 2(4\pi R_2^2 - 4\pi R_1^2) = 8\pi T[R_2^2 - R_1^2]$$

$= 8\pi \times 0.03 \text{ N m}^{-1} \times [(5 \times 10^{-2} \text{ m})^2 - (3 \times 10^{-2} \text{ m})^2]$
$= 3.84\pi \times 10^{-4} \text{ J} \approx 4\pi \times 10^{-4} \text{ J} = 0.4\pi$ mJ

9. **(b):** Work done
= change in pressure × change in volume
$= (2P_0 - P_0)(3V_0 - V_0) = 2P_0V_0$

Since, the cyclic process is anticlockwise, work done by the gas is negative *i.e.*, $-2P_0V_0$.

10 **(b):** Mass per unit length of the string,

$$\mu = \frac{M}{L} = \frac{5\times10^{-4}\ \text{kg}}{1\ \text{m}} = 5\times10^{-4}\ \text{kg m}^{-1}$$

When the wire is plucked at 25 cm length, it vibrates in 2 loops.

Thus,

$$\upsilon = \frac{2}{2L}\sqrt{\frac{T}{\mu}} = \frac{2}{2\times1}\sqrt{\frac{20}{5\times10^{-4}}}\ \text{Hz} = 200\ \text{Hz}$$

11. **(d):** Here, $m = 5$ kg, $p_1 = 10$ kg m s^{-1}, $F = 0.2$ N, $t = 10$ s

Initial kinetic energy $= \dfrac{p_1^2}{2m} = \dfrac{10\times10}{2\times5} = 10$ J

Impulse $= p_2 - p_1 = F\times t$

or $p_2 - 10 = 0.2\times10$

$\Rightarrow$ $p_2 = 12$ kg m s^{-1}

Final kinetic energy $= \dfrac{p_2^2}{2m} = \dfrac{12\times12}{2\times5} = 14.4$ J

Increase in kinetic energy $= 14.4 - 10 = 4.4$ J

12. **(c):** As mass of quarter disc is $\dfrac{M}{4}$, therefore, its moment of inertia about the given axis is

$$I = \frac{1}{2}\left(\frac{M}{4}\right)R^2 = \frac{1}{8}MR^2$$

13. **(b):** The time taken to come back to the same height

$$t_1 = \frac{2u\sin\theta}{g} = \frac{2\times50\times\sin30^\circ}{10} = 5\ \text{s}$$

Taking vertical downward motion of projectile from point of projection to ground, we have

$u = -50\sin30^\circ = -25$ m s^{-1},

$a = 10$ m s^{-2}, $s = 70$ m

$$s = ut + \frac{1}{2}at_2^2$$

$$\therefore\quad 70 = -25\times t_2 + \frac{1}{2}\times10\times t_2^2$$

or $5t^2 - 25t - 70 = 0$ or $t^2 - 5t - 14 = 0$

On solving, we get $t = 7$ s

Hence, $t = t_1 + t_2 = 5 + 7 = 12$ s

14. **(c):** Mass of the bob = 10 g = 0.01 kg

When the bob falls through a vertical height of 1 m, the velocity acquired at the lowest point,

$$\upsilon = \sqrt{2gh} = \sqrt{2\times10\times1} = \sqrt{20}\ \text{m s}^{-1}$$

Centrifugal force $= \dfrac{m\upsilon^2}{r} = \dfrac{0.01\times20}{1} = 0.2$ N

Net tension = weight + centrifugal force

$= (0.01\times10 + 0.20) = 0.3$ N

15 **(d):** When a ray of light moves from one medium to other, its velocity changes. This change depends on refractive index of the medium. Here, light travels from denser to rarer medium, *i.e.*, from medium of higher refractive index to lower refractive index. So, in rarer medium its velocity increases.

16. **(d):** Here, $\lambda = 600$ nm $= 600\times10^{-9}$ m,

$D = 2$ m, $a = 1$ mm $= 10^{-3}$ m

Distance between first dark fringe on either side of central bright fringe = width of central maximum

$$= \frac{2\lambda D}{a} = \frac{2\times(600\times10^{-9})\times2}{10^{-3}}$$

$= 24\times10^{-4}$ m $= 2.4$ mm

17. **(b):** In an electromagnetic wave, the average energy density of magnetic field μ_B = average energy density of electric field $\mu_E = \dfrac{1}{4}\varepsilon_0E_0^2$

$$= \frac{1}{4}\times(8.85\times10^{-12})\times1^2$$

$= 2.2\times10^{-12}$ J m^{-3}

18. **(c):** When four helium nuclei are fused together, one oxygen nucleus is formed. The reaction is

$$4\times{}_2\text{He}^4 \longrightarrow {}_8\text{O}^{16} + Q$$

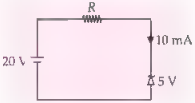
Mass defect, $\Delta m = 4\times m_{\text{He}} - m_{\text{O}_2}$

$= 4\times4.0026 - 15.9994 = 0.011$ amu

Energy produced,

$E = 0.011\times931 = 10.24$ MeV

19. **(d):**



Voltage available across load resistance R

$= 20 - 5 = 15$ V

Resistance of load,

$$R = \frac{15}{10\times10^{-3}} = 1.5\times10^3\ \Omega = 1.5\ \text{k}\Omega$$

20. **(b):** As $I_{rms} = I_0/\sqrt{2}$, $I_0 = I_{rms}\sqrt{2}$, and

$$I = I_0\sin\omega t = I_{rms}\sqrt{2}\sin\left(2\pi\upsilon\times\frac{1}{300}\right)$$

$$= I_{rms}\sqrt{2}\sin(\pi/3) \qquad (\because \upsilon = 50\ \text{Hz})$$

$$= 5\times\sqrt{2}(\sqrt{3}/2) = 5\sqrt{3/2}\ \text{A}$$

21. **(c)** : As $I = \dfrac{200}{\sqrt{R^2 + X_L^2}} = 1$ or $\sqrt{R^2 + X_L^2} = 200$

or $\quad R^2 + X_L^2 = 40000$

$X_L^2 \Rightarrow 40000 - (150)^2$ or $X_L^2 = 17500$

Since $\upsilon' = 100$ Hz and $\upsilon = 50$ Hz, $\therefore\ \upsilon' = 2\upsilon$

As $\quad X_L = 2\pi\upsilon L, \dfrac{X_L'}{X_L} = \dfrac{2\pi\upsilon L}{2\pi\upsilon L} = 2$ or $X_L' = 2X_L$

or $\quad X_L'^2 = 4X_L^2 = 4 \times 17500 = 70000$

$$Z' = \sqrt{R^2 + X_L'^2} = \sqrt{(150)^2 + 70000}\ \Omega = 304\ \Omega$$

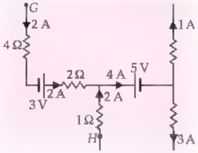
$$I' = \frac{V'}{Z'} = \frac{400\ V}{304\ \Omega} = 1.3\ \text{A}$$

i.e., between 1 A and 2 A

22. **(a)** : When length $(2a)$ of the magnet is much shorter than the distance (r) of the point on the axial line where its magnetic field is to be found,

$$B \propto \frac{1}{r^3} \quad \therefore \quad \frac{B_A}{B_B} = \left(\frac{48\ \text{cm}}{24\ \text{cm}}\right)^3 = 8$$

23. **(c)** : The current distribution in a circuit is as shown in the figure.



Let V_G and V_H be the potentials at points G and H respectively.

$$\therefore \quad V_G - (2\ \text{A})(4\ \Omega) + 3\ \text{V} - (2\ \text{A})(2\ \Omega) + (2\ \text{A})(1\ \Omega) = V_H$$

$$V_G - 8\ \text{V} + 3\ \text{V} - 4\ \text{V} + 2\ \text{V} = V_H$$

$$V_G - V_H = 7\ \text{V}$$

24. **(a)**: According to Einstein's photoelectric equation

$h\upsilon = (\text{KE})_{max} + \phi_0$

But $\phi_0 = h\upsilon_0, \upsilon_0$ being threshold frequency.

$\therefore \quad (\text{KE})_{max} = h\upsilon - h\upsilon_0$

or $\quad (\text{KE})_{max} \propto \upsilon - \upsilon_0$

25. **(c)** : As $\tan\phi = \dfrac{X_L}{R} = \dfrac{\omega L}{R} = \dfrac{2\pi\upsilon L}{R}$

Here, $\upsilon = 50$ Hz, $L = 0.21$ H, $R = 12\ \Omega$

$$\therefore \quad \tan\phi = \frac{2 \times 3.14 \times 50 \times 0.21}{12}$$

$\tan\phi = 5.495$

$\therefore \phi = 80°$

26. **(b)**: As $\beta = \dfrac{\lambda D}{d}$ and $c = \upsilon\lambda$

If frequency is doubled, wavelength becomes half.

27. **(a)** : Here, $m = 2$ kg, $\mu = 0.4$, $g = 10$ m s^{-2}

Minimum force required to move the block

$= \mu R = \mu\, mg = 0.4 \times 2 \times 10 = 8$ N

Since the force applied is only 2.8 N, the block fails to move and static friction

= applied force = 2.8 N

28. **(c)** : By impulse-momentum theorem

$|\vec{F} \times \Delta t| = |\text{change in momentum}|$

Here, $m = 150$ g $= 0.15$ kg, $v = 20$ m s^{-1}; $t = 0.1$ s

$\therefore \quad F \times 0.1 = |\vec{p}_f - \vec{p}_i| \quad$ ... (i)

$p_i = mv_i = 0.15 \times 20 = 3$

As the ball will stop after catching

$\therefore \quad p_f = 0$

Hence, $F \times 0.1 = 3 \quad$ (using (i))

$\therefore \quad F = 30$ N

29. **(a)** : The moment of inertia of a disc about the axis is

$I = \dfrac{1}{2}mR^2$

$$\text{Hence,} \quad \frac{I_1}{I_2} = \frac{\frac{1}{2}M_1R_1^2}{\frac{1}{2}M_2R_2^2} = \frac{\pi R_1^2 td \times R_1^2}{\pi R_2^2 td \times R_2^2}$$

$(\because M = \pi R_1^2 td$ and $M_1 = M_2)$

$$\therefore \quad \frac{I_1}{I_2} = \frac{R_1^4}{R_2^4}$$

$$= \frac{(0.2)^4}{(0.6)^4} = \left(\frac{0.1}{0.3}\right)^4 \quad (\because R_1 = 0.2\ \text{cm}, R_2 = 0.6\ \text{cm})$$

$$= \frac{1}{81}$$

30. **(a)** : Time taken by stone to drop into lake is obtained from

$$s = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2s}{g}}$$

Here, $s = 500$ m, $g = 10$ m s^{-2}, $v = 330$ m s^{-1}

$$\therefore \quad t = \sqrt{\frac{2 \times 500}{10}} = 10\ \text{s}$$

Time taken by the sound produced, to travel to the top of the tower.

$t' = \frac{s}{v} = \frac{500}{330} = 1.5$ s

Total time $= t + t' = 10 + 1.5 = 11.5$ s

31. **(a)**: Force acting on the electron moving in a magnetic field $(\vec{B})$ with velocity $(\vec{v})$,

$\vec{F}_m = e(\vec{v}\times\vec{B}) = e[v_0 i \times B_0 j] = ev_0B_0k$

Since $\vec{F}_m$ (along k) is perpendicular to $\vec{v}$ (along i), it $(\vec{v})$ remains constant.

Thus, de Broglie wavelength, $\lambda = \frac{h}{mv} =$ constant

32. **(c)**: Here, $T_{1/2} = 40$ years, $N = \frac{1}{4}N_0$

As $\frac{N_0}{N} = 2^{t/T_{1/2}}$ or $4 = 2^{t/T_{1/2}}$ or $2^2 = 2^{t/T_{1/2}}$

or $t = 2T_{1/2} = 2 \times 40 = 80$ years

$\therefore \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{40} = 0.0173$ per year

33. **(a)**: If the p-n junction is forward biased, the width of depletion zone decreases. Due to which there will be diffusion across p-n junction due to majority carriers electrons from n side to p-side as if there is increases in number donors on the n-side.

34. **(d)**: As $\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$

$\therefore \frac{\sin\left(\frac{60+\delta_m}{2}\right)}{\sin 30°} = \sqrt{2}$ or $\sin\frac{(60+\delta_m)}{\frac{1}{2}} = \sqrt{2}$

$\sin\frac{(60°+\delta_m)}{2} = \frac{1}{\sqrt{2}} = \sin 45°$

$\Rightarrow \frac{60+\delta_m}{2} = 45°$

$\Rightarrow \delta_m = 30°$

35. **(b)**: ωt and kx both are dimensionless. Out of the given options, only r/y is dimensionless.

36. **(a)**: Here, $u = 10$ m s^{-1}, $t = 3$ s, $v = 16$ m s^{-1}

$\therefore \quad a = \frac{v-u}{t} = \frac{16-10}{3} = 2$ m s^{-2}

Velocity at 2 s before the given instant,

$10 = u + 2 \times 2 \qquad (\because v = u + at)$

$\therefore u = 10 - 4 = 6$ m s^{-1}

37. **(c)**: By conservation of angular momentum,

$\frac{1}{2}MR^2\omega = \frac{1}{2}(M-m)R^2\omega' + mR^2\omega$

or $(M-m)\omega' = (M-2m)\omega$

$\omega' = \left(\frac{M-2m}{M-m}\right)\omega$

38. **(a)**: If r is the distance between Q and $-2Q$, electric field due to $-2Q$ at the location of Q,

i.e. $\quad E = k_e\frac{-(2Q)}{r^2} = -2k_e\frac{Q}{r^2} \qquad \text{... (i)}$

Electric field due to Q at the location of $-2Q$

i.e. $\quad E' = k_e\frac{Q}{r^2} \qquad \text{... (ii)}$

Thus from (i) and (ii), $E' = -E/2$

39. **(c)**: Here, $\varepsilon = 1.5$ V, $r = 2\ \Omega$, $R_{eq} = 5\ \Omega + 8\ \Omega$, $R = 5\ \Omega$

The potential difference across 5Ω resistance,

$$V = \left(\frac{\varepsilon}{R_{eq}+r}\right)R = \left[\frac{1.5\text{ V}}{(5\ \Omega+8\ \Omega)+2\ \Omega}\right]5\ \Omega = 0.5\text{ V}$$

40. **(d)**: Magnetic field $(\vec{B})$ due to the current-carrying solenoid is along its axis, *i.e.*, in the direction of uniform velocity $(\vec{v})$ of the electron.

As $\vec{F}_m = e(\vec{v}\times\vec{B}) = \hat{0}$ (since $\vec{v} \parallel \vec{B}$), the electron continues to move with uniform velocity $\vec{v}$ along the axis of the solenoid.

41. **(c)**: In free expansion, pressure outside is zero. So no work is done by ideal gas but in real gas work may be done against internal forces existing between molecules.

42. **(d)**: Frictional work is not completely recoverable. When the force of friction is present the total mechanical energy is not conserved. The friction force is therefore called a nonconservative or a dissipative force.

The assertion is not true. Frictional force are not conservative.

The reason is not true. It is a false correlation.

43. **(c)**: For a periodic function to be simple harmonic

$\frac{d^2y}{dt^2} \propto -y.$

But given function $y = \sin\omega t + \cos 2\omega t$ will not satisfy the same.

So, it is periodic but not simple harmonic.

44. **(b)**: Cloth can be pulled out without dislodging the dishes from the table because of inertia.

Reason is Newton's third law and hence true. But is not a correct explanation of assertion.

45. **(a):** Electric field is set up from higher potential to lower potential. An electron is negatively charged and moves opposite to the direction of electric field, *i.e.*, from lower potential to higher potential.

46. **(c):** As $R \propto \frac{1}{P}$, resistance of 100 W bulb is less than that of 60 W bulb. Since resistance of 60 W is more, its filament produces more heat and glows more due to higher temperature.
Resistance R is inversely proportional to wattage W, *i.e.*, $R \propto 1/W$.

47. **(a):** As $\varepsilon = \oint \vec{E} \cdot d\vec{l}$ here, $\vec{E}$ is a non-conservative and a time varying field that is generated by a charging magnetic field. This field is quite different from an electrostatic field produced by stationary charges. Here, ε stands for emf and E for electric field.

48. **(d) :** As $qE = \frac{mv^2}{r}$,

$$\Rightarrow \quad r = \frac{mv^2}{qE} = \frac{2K}{qE} \left(\because K = \frac{1}{2}mv^2\right)$$

i.e. $\quad r \propto \frac{1}{q}$

Since electron and proton have equal charge, so r is the same for both of them.

49. **(d):** The position of centre of mass of electron and proton remains at rest, and their motion is due to (internal) forces of electrostatic attraction, which are conservative. No external force, what so ever is acting on the two particles.

50. **(a):** Upto ordinary heights the change in the distance of a projectile from the centre of the earth is negligible compared to the radius of the earth. Hence, projectile moves under a nearly uniform gravitation force and its path is parabolic. But for projectile going to great height, the gravitational force decreases in inverse proportion to the square of the distance of the projectile from the centre of the earth. Under such a variable force the path of projectile is elliptical.

51. **(a):** When roads are not properly banked force of friction between tyres and road provides partially the necessary centripetal force. This causes wear and tear.

52. **(b):** When the bodies are moving in same direction, relative velocity between them is equal to the differences in velocities of two bodies. When two bodies are moving in opposite directions, relative velocity between them is equal to the sum of the velocities of two bodies.

53. **(a):** As $\frac{1}{f} = \left(\frac{\mu_g}{\mu_a} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$;

$$\frac{1}{10} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{5}$$

If f_w is the focal length of the lens when immersed in water,

$$\frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{5}\right) = \frac{1}{40}$$

Hence, $f_w = 40$ cm

54. **(a):** Photoelectric effect is a one-to-one process, *i.e.*, only one photon interacts with one electron.

55. **(a):** ${}^{90}_{38}\text{Sr}$ decays to ${}^{90}_{39}\text{Y}$ through β-emission $\left({}^{90}_{38}\text{Sr} \rightarrow {}^{90}_{39}\text{Y} + {}^{0}_{-1}e\right)$.

Bone marrow is damaged by these high energy β particles.

56. **(c):** The collector in common base configuration is reverse biased for voltage amplification. Simply stating amplification may include current amplification (which is not so in this case).

57. **(a):** Since pressure inside bubble of smaller radius is more the common interface should bulge inside bubble of larger radii. Hence reason is correct explanation of assertion.

58. **(a):** When external torque acting on the system is zero then only the total angular momentum of system is considered to be conserved.

$$\vec{\tau} = \frac{d\vec{L}}{dt},$$

If $\vec{\tau} = 0$, $\frac{d\vec{L}}{dt} = 0 \quad \Rightarrow \vec{L} = I\omega = \text{constant}$

59. **(c):** Due to electric field, the force is $\vec{F} = q\vec{E}$ in the direction of $\vec{E}$. Since $\vec{E}$ is parallel to $\vec{B}$, the particle velocity $\vec{v}$ (acquired due to force $\vec{F}$) is parallel to $\vec{B}$. Hence, $\vec{B}$ will not exert any force since $\vec{v} \times \vec{B} = 0$ and thus the motion of the particle is not affected by $\vec{B}$.

60. **(a):** A rocket moves forward taking the help of reaction force. For that it has to exert a force on the surrounding air so that it receives reaction force as per Newton's third law.

NCERT Xtract

New

Questions for Medical/ Engineering Entrance Exams

Electromagnetic Induction

1. Two coils A and B are separated by a certain distance. If a current of 4 A flows through A, a magnetic flux of 10^{-3} Wb passes through B (no current through B). If no current passes through A and a current of 2 A passes through B, then the flux through A is
   (a) 5×10^{-3} Wb (b) 4×10^{-4} Wb
   (c) 5×10^{-4} Wb (d) 2×10^{-3} Wb

2. A coil of area 0.4 m^2 has 100 turns. A magnetic field of 0.04 Wb m^{-2} is acting normal to the coil surface. If this magnetic field is reduced to zero in 0.01 s, then the induced emf in the coil is
   (a) 160 V (b) 250 V
   (c) 270 V (d) 320 V

3. The magnetic flux linked with a coil of N turns of area of cross section A held with its plane parallel to the field B is
   (a) $\dfrac{NAB}{2}$ (b) NAB
   (c) $\dfrac{NAB}{4}$ (d) zero

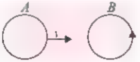
4. A jet plane is travelling west at the speed of 1600 km h^{-1}. The voltage difference developed between the ends of the wing having a span of 20 m, if the earth's magnetic field at the location has a magnitude of 5×10^{-4} T and the dip angle is 30° is
   (a) 4.1 V (b) 2.2 V
   (c) 3.2 V (d) 3.8 V

5. A long solenoid with 10 turns per cm has a small loop of area 3 cm^2 placed inside, normal to the axis of the solenoid. If the current carried by the solenoid changes steadily from 2 A to 4 A in 0.2 s, what is the induced voltage in the loop, while the current is changing?
   (a) 4.2×10^{-8} V (b) 2.8×10^{-8} V
   (c) 7.3×10^{-6} V (d) 3.8×10^{-6} V

6. A square loop of side 12 cm and resistance 0.60 Ω is placed vertically in the east-west plane. A uniform magnetic field of 0.10 T is set up across the plane in north-east direction. The magnetic field is decreased to zero in 0.6 s at a steady rate. The magnitude of current during this time interval is
   (a) 1.42×10^{-3} A (b) 2.67×10^{-3} A
   (c) 3.41×10^{-3} A (d) 4.21×10^{-3} A

7. A magnetic flux through a stationary loop with a resistance R varies during the time interval τ as $\phi = at(\tau - t)$. What is the amount of heat generated in the loop during that time ?
   (a) $\dfrac{a^2\tau^3}{4R}$ (b) $\dfrac{a^2\tau^3}{3R}$
   (c) $\dfrac{a^2\tau^3}{6R}$ (d) $\dfrac{a^2\tau^3}{2R}$

8. Two coils, A and B are as shown in the figure. A current starts flowing in B as shown, when A is moved toward B and stops when A stops moving.



The current in A is counterclockwise. B is kept stationary when A moves. We can infer that

(a) there is a constant current in the clockwise direction of A.
(b) there is a no current in A.
(c) there is a varying current in A.
(d) there is a constant current in the counterclockwise direction in A

9. Two solenoids of equal number of turns have their lengths and the radii in the same ratio 1 : 2. The ratio of their self inductances will be

(a) 1 : 2 (b) 2 : 1
(c) 1 : 1 (d) 1 : 4

10. If the number of turns per unit length of a coil of solenoid is doubled, the self-inductance of the solenoid will

(a) remain unchanged (b) be halved
(c) be doubled (d) become four times

11. A circular coil of radius 6 cm and 20 turns rotates about its vertical diameter with an angular speed of 40 rad s^{-1} in a uniform horizontal magnetic field of magnitude 2×10^{-2} T. If the coil form a closed loop of resistance 8 Ω, then the average power loss due to joule heating is

(a) 2.07×10^{-3} W (b) 1.23×10^{-3} W
(c) 3.14×10^{-3} W (d) 1.80×10^{-3} W

12. A pair of adjacent coils has a mutual inductance of 2.5 H. If the current in one coil changes from 0 to 40 A in 0.8 s, then the change in flux linked with the other coil is

(a) 100 Wb (b) 120 Wb
(c) 200 Wb (d) 250 Wb

13. A wheel with 20 metallic spokes each of length 0.8 m long is rotated with a speed of 120 revolution per minute in a plane normal to the horizontal component of earth magnetic field H at a place. If $H = 0.4 \times 10^{-4}$ T at the place, then induced emf between the axle and the rim of the wheel is

(a) 2.3×10^{-4} V (b) 3.1×10^{-4} V
(c) 2.9×10^{-4} V (d) 1.61×10^{-4} V

14. A metal conductor of length 1 m rotates vertically about one of its ends with an angular velocity 5 rad s^{-1}. If the horizontal component of earth's magnetic field is 0.2×10^{-4} T, then the emf developed between the ends of the conductor is

(a) 5 μV (b) 5 mV
(c) 50 μV (d) 50 mV

15. Two conducting circular loops of radii R_1 and R_2 are placed in the same plane with their centres coinciding. If $R_1 > R_2$ the mutual inductance M between them will be directly proportional to

(a) $\frac{R_1}{R_2}$ (b) $\frac{R_2}{R_1}$
(c) $\frac{R_1^2}{R_2}$ (d) $\frac{R_2^2}{R_1}$

16. A rectangular loop of sides 6 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.4 T directed normal to the loop. The voltage developed across the cut if velocity of loop is 2 cm s^{-1} in a direction normal to the longer side is

(a) 3.8×10^{-4} V (b) 4.8×10^{-4} V
(c) 2.2×10^{-2} V (d) 3.2×10^{-2} V

17. A small flat search coils of area 4 cm^2 with 20 closely wound turns is positioned normal to the field direction and then quickly snatched out of the field region. The total charge flowing in the coil (measured by a ballistic galvanometer connected to the coil) is 7.5 mC. The resistance of the coil and galvanometer is 0.8 Ω. The field strength of the magnet is

(a) 1.25 T (b) 0.50 T
(c) 0.75 T (d) 2.10 T

18. A conducting ring of radius r is placed in a varying magnetic field perpendicular to the plane of the ring. If the rate at which the magnetic field varies is x, the electric field intensity at any point of the ring is

(a) rx (b) $\frac{rx}{2}$
(c) $2rx$ (d) $\frac{4r}{x}$

19. An air cored solenoid with length 20 cm, area of cross section 20 cm^2 and number of turns 400 carries a current 2 A. The current is suddenly switched off within 10^{-3} s. The average back emf induced across the ends of the open switch in the circuit is (ignore the variation in magnetic field near the ends of the solenoid)

(a) 2 V (b) 4 V
(c) 3 V (d) 5 V

20. A rod of length l rotates with a uniform angular velocity ω about an axis passing through its middle point but normal to its length in a uniform magnetic field of induction B with its direction parallel to the axis of rotation. The induced emf between the two ends of the rod is

(a) $\frac{Bl^2\omega}{2}$ (b) zero

(c) $\left(\frac{Bl^2\omega}{8}\right)$ (d) $2Bl^2\omega$

21. The magnetic flux through a coil perpendicular to its plane and directed into paper is varying according to the relation $\phi = (2t^2 + 4t + 6)$ mWb. The emf induced in the loop at $t = 4$ s is

(a) 0.12 V (b) 2.4 V
(c) 0.02 V (d) 1.2 V

22. A 2 m long solenoid with diameter 2 cm and 2000 turns has a secondary coil of 1000 turns wound closely near its midpoint. The mutual inductance between the two coils is

(a) 2.4×10^{-4} H (b) 3.9×10^{-4} H
(c) 1.28×10^{-3} H (d) 3.14×10^{-3} H

23. A 10 V battery connected to 5 Ω resistance coil having inductance 10 H through a switch drives a constant current in the circuit. The switch is suddenly opened and the time taken to open it is 2 ms. The average emf induced across the coil is

(a) 4×10^4 V (b) 2×10^4 V
(c) 2×10^2 V (d) 1×10^4 V

24. There is a uniform magnetic field directed perpendicular and into the plane of the paper An irregular shaped conducting loop is slowly changing into a circular loop in the plane of the paper. Then

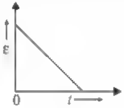
(a) current is induced in the loop in the anti-clockwise direction
(b) current is induced in the loop in the clockwise direction.
(c) ac is induced in the loop.
(d) no current in induced in the loop.

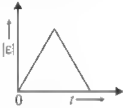
25. The figure shows a wire sliding on two parallel conducting rails placed at a separation l. A magnetic field B exists in a direction perpendicular to the plane of the rails. The force required to keep the wire moving at a constant velocity v will be

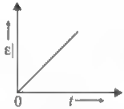
(a) evB (b) $\frac{\mu_0 Bv}{4\pi l}$

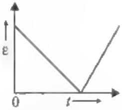
(c) Blv (d) zero

26. A long solenoid S has n turns per metre, with diameter a. At the centre of this coil, we place a smaller coil of N turns and diameter b ($b < a$). If the current in the solenoid increases linearly with time, then the emf will be induced in the smaller coil. Which of the following is the correct graph showing $|\varepsilon|$ verses t if current varies as a function of $mt^2 + C$?

(a) 

(b) 

(c) 

(d) 

27. A magnetic field B is confined to a region $r \leq a$ and points out of the paper (the z-axis), $r = 0$ being the centre of the circular region. A charged ring (charge = q) of radius $b(b > a)$ and mass m lies in the x-y plane with its centre at the origin. The ring is free to rotate and is at rest. The magnetic field is brought to zero in time Δt. The angular velocity ω of the ring after the field vanishes, is

(a) $\frac{qBa^2}{2mb}$ (b) $\frac{qBa}{2mb^2}$

(c) $\frac{2mb^2}{qBa^2}$ (d) $\frac{qBa^2}{2mb^2}$

28. In a uniform magnetic field B a wire in the form of a semicircle of radius r rotates about the diameter of the circle with angular frequency ω. The axis of rotation is perpendicular to the field. If the total resistance of the circuit is R, the mean power generated per period of rotation is

(a) $\frac{B\pi r^2\omega}{2R}$ (b) $\frac{(B\pi r^2\omega)^2}{8R}$

(c) $\frac{(B\pi r\omega)^2}{2R}$ (d) $\frac{(B\pi r\omega^2)^2}{8R}$

29. The self inductance of a coil having 400 turns is 10 mH. The magnetic flux through the cross section of the coil corresponding to current 2 mA is

(a) 4×10^{-5} Wb (b) 2×10^{-3} Wb
(c) 3×10^{-5} Wb (d) 8×10^{-3} Wb

30. A copper rod of length l rotates about its end with angular velocity ω in a uniform magnetic field B. The emf developed between the ends of the rod if the field is normal to the plane of rotation is

(a) $B\omega l^2$ (b) $\frac{1}{2}B\omega l^2$

(c) $2\ B\omega l^2$ (d) $\frac{1}{4}B\omega l^2$

31. A current of 1 A through a coil of inductance of 200 mH is increasing at a rate of 0.5 A s^{-1}. The energy stored in the inductor per second is

(a) 0.5 J s^{-1} (b) 5.0 J s^{-1}

(c) 0.1 J s^{-1} (d) 2.0 J s^{-1}

32. An electromagnet has stored 648 J of magnetic energy when a current of 9 A exists in its coils. What average emf is induced if the current is reduced to zero in 0.45 s?

(a) 320 V (b) 620 V

(c) 260 V (d) 230 V

33. A circular disc of radius 0.2 m is placed in a uniform magnetic field of induction $\left(\frac{1}{\pi}\right)$Wb m^{-2} in such a way that its axis makes an angle of 60° with $\vec{B}$. The magnetic flux linked with the disc is

(a) 0.02 Wb (b) 0.06 Wb

(c) 0.08 Wb (d) 0.01 Wb

34. A solenoid of length 30 cm with 10 turns per centimetre and area of cross-section 40 cm^2 completely surrounds another co-axial solenoid of same length, area of cross-section 20 cm^2 with 40 turns per centimetre. The mutual inductance of the system is

(a) 10 H (b) 8 H

(c) 3 mH (d) 30 mH

35. By a change of current from 5 to 10 amperes in 0.1 second, the self induced emf is 10 V. The change in the energy of the magnetic field of a coil will be

(a) 5 J (b) 6 J

(c) 7.5 J (d) 9 J

SOLUTION

1. **(c)** : Magnetic flux linked with coil

$$\phi_B = MI_A$$

$$\therefore \quad M = \frac{\phi_B}{I_A} = \frac{10^{-3}}{4} \text{ henry}$$

Magnetic flux linked with coil A

$$\phi_A = MI_B = \frac{10^{-3}}{4} \times 2 = 5 \times 10^{-4} \text{ Wb}$$

2. **(a)** : Here, $A = 0.4$ m^2, $N = 100$, $dB = 0.04$ Wb m^{-2}, $dt = 0.01$ s

As $\varepsilon = \frac{d\phi}{dt} = NA\frac{dB}{dt}$ $(\because \phi = BA)$

$= 100 \times 0.4 \times \frac{0.04}{0.01} = 160$ V

3. **(d)**: Magnetic flux linked with a coil

$\phi = NBA\cos\theta$

Since the magnetic field B is parallel to the area A, *i.e.*, $\theta = 90°$.

$\therefore$ $\phi = 0$

4. **(b)**: Here, $v = 1600$ km h^{-1} $= 1600 \times \frac{5}{18}$ m s^{-1}

$= 444$ m s^{-1}

$l = 20$ m , $B = 5 \times 10^{-4}$ T and $\delta = 30°$

As $\varepsilon = Blv = Vlv$

(where V is the vertical component of earth's field)

$= (B \sin\delta)\ lv$ $(\because V = B\sin\delta)$

$= 5 \times 10^{-4} \sin 30° \times 20 \times 444$

$= 2.2$ V

5. **(d)** : Here, $\frac{N}{l} = 10$ turns per cm $= 1000$ turns per m

$A = 3$ cm^2 $= 3 \times 10^{-4}$ m^2

or $\frac{dI}{dt} = \frac{4-2}{0.2} = 10$ A s^{-1}

Also $\varepsilon = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = A\frac{d}{dt}\left(\mu_0\frac{N}{l}I\right)$

$= A\mu_0\left(\frac{N}{l}\right)\frac{dI}{dt}$

$= 3 \times 10^{-4} \times 4\pi \times 10^{-7} \times 1000 \times 10$

$= 3.8 \times 10^{-6}$ V

6. **(b)**: Here, Area $A = l^2 = (12\text{ cm})^2 = 1.4 \times 10^{-2}$ m^2

$R = 0.60\ \Omega$, $B_1 = 0.10$ T, $\theta = 45°$

$B_2 = 0$, $dt = 0.6$ s

Initial flux, $\phi_1 = B_1A\cos\theta$

$= 0.10 \times 1.4 \times 10^{-2} \times \cos 45°$

$= 9.9 \times 10^{-4}$ Wb

Final flux, $\phi_2 = 0$

Induced emf, $\varepsilon = \frac{|d\phi|}{dt} = \frac{|\phi_2 - \phi_1|}{dt}$

$$= \frac{|9.8 \times 10^{-4}|}{0.6\ \text{s}} = 1.65 \times 10^{-3}\ \text{V}$$

Current, $I = \frac{\varepsilon}{R} = \frac{1.65 \times 10^{-3}}{0.6}$

$= 2.5 \times 10^{-3}$ A

7. **(b):** The flux through the stationary loop is
$\phi = at(\tau - t)$
Induced emf,
$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}[at(\tau - t)] = -[a\tau - 2at] = (2at - a\tau)$$
The amount of heat generated in the loop during a small time interval dt is
$$dQ = \frac{\varepsilon^2}{R}dt = \frac{(2at - a\tau)^2}{R}dt$$
Hence, the total amount of heat generated is
$$Q = \int_0^\tau \frac{(2at - a\tau)^2}{R}dt = \frac{1}{R}\int_0^\tau (4a^2t^2 + a^2\tau^2 - 4a^2\tau t)dt$$
$$= \frac{1}{R}\left[\frac{4}{3}a^2t^3 + a^2\tau^2 t - \frac{4}{2}a^2\tau t^2\right]_0^\tau = \frac{1}{3}\frac{a^2\tau^3}{R}$$

8. **(d):** Coil A must be carrying a constant current in counter clockwise direction. Because of that, when A moves towards B, current induced in B is counterclockwise direction as per Lenz's law. The current in B would stop when A stops moving.

9. **(a).** Self inductance of a solenoid,
$$L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 N^2 \pi r^2}{l}$$
where l is the length of the solenoid, N is the total number of turns of the solenoid and A is the area of cross-section of the solenoid
$$\therefore \frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2\left(\frac{r_1}{r_2}\right)^2\left(\frac{l_2}{l_1}\right)$$
Here, $N_1 = N_2$, $\frac{l_1}{l_2} = \frac{1}{2}$, $\frac{r_1}{r_2} = \frac{1}{2}$
$$\therefore \frac{L_1}{L_2} = \left(\frac{1}{2}\right)^2\left(\frac{2}{1}\right) = \frac{1}{2}$$

10. **(d):** Self inductance of a solenoid $= \mu_0 n^2 A l$
where n is the number of turns per length.
$\therefore$ Self inductance is directly proportional to n^2. Self inductance becomes 4 times when n is doubled.

11. **(a):** Here, $r = 6$ cm $= 6 \times 10^{-2}$ m, $N = 20$,
$\omega = 40$ rad s^{-1}
$B = 2 \times 10^{-2}$ T, $R = 8\ \Omega$
Maximum emf induced, $\varepsilon = NAB\omega$
$= N(\pi r^2)B\omega$
$= 20 \times \pi \times (6 \times 10^{-2})^2 \times 2 \times 10^{-2} \times 40$
$= 0.18$ V
Average value of emf induced over a full cycle
$\varepsilon_{av} = 0$

Maximum value of current in the coil,
$$I = \frac{\varepsilon}{R} = \frac{0.18}{8} = 0.023\text{ A}$$
Average power dissipated,
$$P = \frac{\varepsilon I}{2} = \frac{0.18 \times 0.023}{2} = 2.07 \times 10^{-3}\text{ W}$$

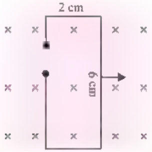
12. **(a):** Given, $M = 2.5$ H, $\frac{dI}{dt} = \frac{40 - 0}{0.8} = 50$ A s^{-1}
Also, $\varepsilon = M\frac{dI}{dt} = \frac{d\phi}{dt}$
or $d\phi = MdI = 2.5\ (40 - 0) = 100$ Wb

13. **(d):** Here, $H = B = 0.4 \times 10^{-4}$ T, $l = 0.8$ m
$\upsilon = 120$ rpm $= 2$ rps
Emf induced across the ends of each spoke
$$\varepsilon = \frac{1}{2}B\omega l^2$$
$$= \frac{1}{2}B(2\pi\upsilon)l^2 \qquad (\because\ \omega = 2\pi\upsilon)$$
$= B\pi\upsilon l^2$
$\therefore\ \varepsilon = 0.4 \times 10^{-4} \times \pi \times 2 \times (0.8)^2$
$= 1.61 \times 10^{-4}$ V
Note : Number of spokes is not relevant because the emfs across the spoke area is parallel.

14. **(c) :** The emf developed between the ends of the conductor is
$$\varepsilon = \frac{1}{2}\omega Bl^2 = \frac{1}{2} \times 5 \times 0.2 \times 10^{-4} \times (1)^2$$
$= 5 \times 10^{-5}$ V $= 50 \times 10^{-6}$ V $= 50\ \mu$V

15. **(d):** Let a current I_1 flows through the outer circular coil of radius R_1.
The magnetic field at the centre of the coil is
$$B_1 = \frac{\mu_0 I_1}{2R_1}$$
As the inner coil of radius R_2 placed co-axially has small radius ($R_2 < R_1$), therefore B_1 may be taken constant over its cross-sectional area.
Hence, flux associated with the inner coil is
$$\phi_2 = B_1\pi R_2^2 = \frac{\mu_0 I_1}{2R_1}\pi R_2^2$$
As $M = \frac{\phi_2}{I_1} = \frac{\mu_0 \pi R_2^2}{2R_1}$
$$\therefore\ M \propto \frac{R_2^2}{R_1}$$

16. **(b)**: Here, l = 6 cm = 6×10^{-2} m



B = 0.4 T,
v = 2 cm s^{-1} = 2×10^{-2} m s^{-1}
Voltage developed is
$\varepsilon = Blv = 0.4 \times 6 \times 10^{-2} \times 2 \times 10^{-2}$
$= 4.8 \times 10^{-4}$ V

17. **(c)** : Here, A = 4 cm^2 = 4×10^{-4} m^2, N = 20
Final flux, $\phi_f = 0$ (when the coil is removed from the field)
q = 7.5 mC = 7.5×10^{-3} C, $R = 0.8\ \Omega$

As $I = \dfrac{\varepsilon}{R} = \dfrac{-N(d\phi/dt)}{R}$

$$I\,dt = -\frac{N}{R}d\phi$$

Also charge,

$$q = \int I\,dt = \int_{\phi_i}^{\phi_f} -\frac{N}{R}d\phi = -\frac{N}{R}(\phi_f - \phi_i) = \frac{N}{R}(\phi_i - \phi_f)$$

$$q = \frac{N}{R}\phi_i$$

Now, initial flux per turn when coil is normal to the field, $\phi_i = BA$

$\therefore \quad q = \dfrac{NBA}{R}$

or $\quad B = \dfrac{qR}{NA} = \dfrac{7.5 \times 10^{-3} \times 0.8}{20 \times 4 \times 10^{-4}} = 0.75$ T

18. **(b)**: Let $\vec{E}$ be the electric field intensity at a point on the circumference of the ring. Then, the emf induced $\varepsilon = \oint \vec{E} \cdot d\vec{l}$ where $d\vec{l}$ is a length element of the ring. Since $|\vec{E}|$ is constant and $\vec{E} \parallel d\vec{l}$,

$\varepsilon = E(2\pi r)$ ...(i)

Also, the induced emf is

$\varepsilon = \dfrac{d\phi}{dt} = \pi r^2 \dfrac{dB}{dt} = \pi r^2 x$ ...(ii)

Equating (i) and (ii), we get

$$E = \frac{rx}{2}$$

19. **(b)**: Here, l = 20 cm = 20×10^{-2} m,
A = 20 cm^2 = 20×10^{-4} m^2
$N = 400$, $I_1 = 2$ A, $I_2 = 0$, $dt = 10^{-3}$ s

As $\quad \varepsilon = \dfrac{d\phi}{dt} = \dfrac{d(BAN)}{dt}$

$= \dfrac{\mu_0 N\,dI\,AN}{l\,dt} \qquad \left(\because\ B = \dfrac{\mu_0 NI}{l}\right)$

$= \dfrac{\mu_0 N(I_1 - I_2)AN}{l\,dt}$

$= \dfrac{4\pi \times 10^{-7} \times (400)^2 \times 2 \times 20 \times 10^{-4}}{20 \times 10^{-2} \times 10^{-3}} = 4$ V

20. **(b)**: Length of the rod between the axis of rotation and one end of the rod $= \dfrac{l}{2}$

Area swept out in one rotation

$= \pi\left(\dfrac{l}{2}\right)^2 = \left(\dfrac{\pi l^2}{4}\right)$

Angular velocity = ω rad s^{-1}

Frequency of revolution $= \dfrac{\omega}{2\pi}$

Area swept out per second

$= \dfrac{\pi l^2}{4}\left(\dfrac{\omega}{2\pi}\right) = \dfrac{l^2\omega}{8}$

Magnetic induction = B

Rate of change of magnetic flux $= \left(\dfrac{Bl^2\omega}{8}\right)$

Magnitude of induced emf $= \dfrac{Bl^2\omega}{8}$

Magnitude of induced emf between the axis and the other end is also $\left(\dfrac{Bl^2\omega}{8}\right)$.

These two emf's are in opposite directions. Hence, the potential difference between the two ends of the rod is zero.

21. **(c)** : Given, $\phi = (2t^2 + 4t + 6)$ mWb

As, $\quad \varepsilon = \dfrac{d\phi}{dt}$

$= \dfrac{d}{dt}(2t^2 + 4t + 6) \times 10^{-3}$ Wb s^{-1}

$= (4t + 4) \times 10^{-3}$ V

At t = 4 s

$\varepsilon = (4 \times 4 + 4) \times 10^{-3}$ V

$= 20 \times 10^{-3}$ V = 0.02 V

22. (b): Here, $l = 2$ m, diameter = 2 cm

radius, $r = \frac{2}{2} = 1\,\text{cm} = 1\times10^{-2}$ m

$N_1 = 2000$, $N_2 = 1000$

Area $= \pi r^2 = \pi \times (1 \times 10^{-2})^2$
$= 3.14 \times 10^{-4}$ m^2

Mutual inductance, $M = \frac{\mu_0 N_1 N_2 A}{l}$

$$= \frac{4\pi\times10^{-7}\times2000\times1000\times3.14\times10^{-4}}{2} = 3.9\times10^{-4}\ \text{H}$$

23. (d): Here, current $I = \frac{V}{R} = \frac{10}{5} = 2$ A

Final current, when the switch is opened is zero

$$\therefore \quad \frac{dI}{dt} = \frac{0-2}{2\times10^{-3}} = -1\times10^{3}\ \text{A s}^{-1}$$

As $\quad \varepsilon = -L\frac{dI}{dt}$

$= -10\,(-1 \times 10^3) = 10^4$ V

24. (a): Due to change in the shape of the loop, the magnetic flux linked with the loop increases. Hence, current is induced in the loop in such a direction that it opposes the increases in flux. Therefore, induced current flows in the anticlockwise direction.

25. (d): No change in flux, hence no force required.

26. (c): As per the data given in the question, magnetic field due to current in solenoid S, $B = \mu_0 nI$

Magnetic flux linked with the smaller coil due to this field is

$\phi = NBA$, where A = area of smaller coil $= \frac{\pi b^2}{4}$

$\therefore$ emf induced in the smaller coil,

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(NB\frac{\pi b^2}{4}\right)$$

$$= -\frac{N\pi b^2}{4}\frac{dI}{dt}(\mu_0 nI) = -\frac{N\pi b^2}{4}\mu_0\frac{dI}{dt}$$

As current I varies as a function of $(mt^2 + C)$

$$\therefore \quad \varepsilon = -N n\mu_0\frac{\pi b^2}{4}\frac{d}{dt}(mt^2 + C)$$

$$= -\frac{Nn\pi\mu_0 b^2}{4}(2mt) \qquad \text{...(i)}$$

From (i), $|\varepsilon| \propto t$

So correct option is (c).

27. (d): Let E is the electric field generated around the charged ring of radius b, then

$$\varepsilon = \frac{d\phi}{dt}$$

$$\oint \vec{E}\cdot d\vec{l} = \frac{B\pi a^2}{\Delta t}$$

$$\text{or} \quad Eb = \frac{Ba^2}{2(\Delta t)} \qquad \text{..(i)}$$

Torque acting on the ring

$\tau = b \times \text{force} = bqE$

$$= \frac{qBa^2}{2(\Delta t)} \qquad \text{[Using (i)]}$$

If ΔL is change in angular momentum of the charged ring, then

$$\tau = \frac{\Delta L}{\Delta t} = \frac{L_2 - L_1}{\Delta t}$$

$\therefore \quad L_2 - L_1 = \tau(\Delta t)$

$$= \frac{qBa^2\Delta t}{2\Delta t} = \frac{qBa^2}{2}$$

As initial angular momentum, $L_1 = 0$

$$\therefore \quad L_2 = \frac{qBa^2}{2} = I\omega = mb^2\omega$$

$$\therefore \quad \omega = \frac{qBa^2}{2mb^2}$$

28. (b): $\phi = B\frac{\pi r^2}{2}\cos\omega t$

$$\therefore \quad \varepsilon = -\frac{d\phi}{dt} = \frac{1}{2}B\pi r^2\omega\sin\omega t$$

$$\therefore \quad P = \frac{\varepsilon^2}{R} = \frac{B^2\pi^2 r^4\omega^2\sin^2\omega t}{4R}$$

$$\therefore \quad \langle P\rangle = \frac{(B\pi r^2\omega)^2}{8R} \qquad \left(\because \ \langle\sin^2\omega t\rangle = \frac{1}{2}\right)$$

29. (a): Here, $N = 400$, $L = 10$ mH $= 10 \times 10^{-3}$ H

$I = 2$ mA $= 2 \times 10^{-3}$ A

Total magnetic flux linked with the coil,

$\phi = NLI = 400 \times (10 \times 10^{-3}) \times 2 \times 10^{-3} = 8 \times 10^{-3}$ Wb

Magnetic flux through the cross-section of the coil = Magnetic flux linked with each turn

$$= \frac{\phi}{N} = \frac{8\times10^{-3}}{200} = 4 \times 10^{-5}\ \text{Wb}$$

30. (b)

31. (c) : The energy stored in the inductor is

$$U = \frac{1}{2}LI^2$$

The energy stored in the inductor per second is

$$\frac{dU}{dt} = LI\frac{dI}{dt}$$

$$= (200 \times 10^{-3}\,\text{H})(1\,\text{A})(0.5\,\text{As}^{-1}) = 0.1\,\text{J s}^{-1}$$

32. (a) : Magnetic energy, $U = \frac{1}{2}LI^2$

$$\therefore \quad L = \frac{2U}{I^2} = \frac{2 \times 648}{(9)^2} = 16\,\text{H}$$

Induced emf,

$$\varepsilon = -L\frac{dI}{dt} = \frac{-(16\,\text{H})(0-9\,\text{A})}{0.45\,\text{s}} = 320\,\text{V}$$

33. (a) : Here, $r = 0.2$ m, $B = \frac{1}{\pi}\,\text{Wb m}^{-2}$,

$\theta = 60°$

$\therefore$ Magnetic flux,

$\phi = BA\cos\theta = B(\pi r^2)\cos\theta$

$$= \frac{1}{\pi}\pi(0.2)^2\cos 60° = 0.02\,\text{Wb}$$

34. (c) : Mutual inductance of the system,

$M = \mu_0 n_1 n_2 A_2 l$

where A_2 is the area of inner solenoid.

$n_1 = 10\,\text{cm}^{-1} = 1000\,\text{m}^{-1}$

$n_2 = 40\,\text{cm}^{-1} = 4000\,\text{m}^{-1}$

$l = 30\,\text{cm} = 30 \times 10^{-2}\,\text{m}$

$A_2 = 20\,\text{cm}^2 = 20 \times 10^{-4}\,\text{m}^2$

$\therefore \; M = 4\pi \times 10^{-7} \times 1000 \times 4000 \times 20 \times 10^{-4} \times 30 \times 10^{-2}$

$= 301.44 \times 10^{-5}\,\text{H} = 3\,\text{mH}$

35. (c) : $|\varepsilon| = L\frac{\Delta I}{\Delta t}$

$$L = \frac{|\varepsilon|\Delta t}{\Delta I} = \frac{10 \times 0.1}{(10-5)} = 0.2\,\text{H}$$

The magnetic field energies for currents I_1 and I_2 are

$$U_1 = \frac{1}{2}LI_1^2 \text{ and } U_2 = \frac{1}{2}LI_2^2$$

Change in energy $= U_2 - U_1$

$$= \frac{1}{2}LI_2^2 - \frac{1}{2}LI_1^2 = \frac{L}{2}\left(I_2^2 - I_1^2\right) = \frac{0.2}{2}(10^2 - 5^2) = 7.5\,\text{J}$$

BRAIN MAP: MODERN PHYSICS

Dual Nature of Matter and Radiation

Photon

- Energy of a photon
  $E = h\nu = \frac{hc}{\lambda}$
- Number of photons emitted per sec,
  $N = \frac{P}{E}$
- Momentum of photon,
  $p = \frac{h\nu}{c} = \frac{h}{\lambda}$
- Equivalent mass of a photon,
  $m = \frac{h\nu}{c^2}$

Photoelectric Effect

- Work function, $\phi_0 = h\nu_0 = \frac{hc}{\lambda_0}$
- K.E. of emitted photoelectron
  $K = \frac{1}{2}mv^2 = eV_0 = h\nu - \phi_0$
- If V_0 is the stopping potential, the maximum kinetic energy of the ejected photoelectron,
  $K_{max} = \frac{1}{2}mv_{max}^2 = eV_0$
  $= h\nu - h\nu_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$
- Intensity of radiation $= \frac{\text{Energy}}{\text{Area} \times \text{Time}} = \frac{\text{Power}}{\text{Area}}$
  Incident power
  = Incident intensity × Area
- Wavelength shift in Compton scattering
  $\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\phi)$

de-Broglie Hypothesis

- de-Broglie wavelength, $\lambda = \frac{h}{mv} = \frac{h}{p}$
- Kinetic energy,
  $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$
  ∴ Momentum, $p = \sqrt{2mK}$
- de-Broglie wavelength of an electron beam accelerated through a potential difference of V volt is
  $\lambda = \frac{h}{\sqrt{2meV}} = \frac{1.23}{\sqrt{V}}$ nm

Lasers

- Laser is a source of highly directional, monochromatic and coherent light.
- The action of a laser is based on the principle of stimulated emission of light.
- If the emitted radiation falls in the microwave region the device is termed as a Maser
- Number of atoms in excited state x,
  $N_x = N_0 e^{-(E_x - E_0)/(kT)}$
  where E_0 is energy of atom in ground state k is Boltzmann constant

Atoms

Distance of Closest Approach and Impact Parameter

- Kinetic energy of α-particle,
  $K = \frac{1}{2}mv_\alpha^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{r_0}$
- Distance of closest approach
  $r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{K} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4Ze^2}{mv^2}$
- Impact parameter, $b = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2 \cot\frac{\theta}{2}}{K}$
  $= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2 \cot\frac{\theta}{2}}{\frac{1}{2}mv^2}$

Bohr's Theory of Hydrogen Atom

- $\frac{kZe^2}{r^2} = \frac{mv^2}{r}$
- $mvr = \frac{nh}{2\pi}$
- $h\nu = E_2 - E_1$
- $r_n = \frac{n^2h^2}{4\pi^2 mkZe^2}$
- $v = \frac{2\pi kZe^2}{nh} = \frac{c}{n}\left(\frac{2\pi kZe^2}{ch}\right) = \frac{c}{n}\alpha$
  where, $\alpha = \frac{2\pi kZe^2}{ch}$ is fine structure constant
- $E = \frac{2\pi^2 mk^2Z^2e^4}{n^2h^2} = \frac{mc^2}{2n^2}\alpha^2$
- K.E $= \frac{kZe^2}{2r}$, P.E. $= -\frac{kZe^2}{r}$
- Total energy, $E_n = \frac{-kZe^2}{2r} = \frac{-Rhc}{n^2} = \frac{-13.6}{n^2}$ eV
- Wave number, $\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$
  Where, $R = \frac{2\pi^2 mk^2e^4}{ch^3}$ = Rydberg's constant
  $= 1.097 \times 10^7\ m^{-1}$
  Here, $k = \frac{1}{4\pi\varepsilon_0}$

Pauli Exclusion Principle

- No two electrons can occupy the same quantum-mechanical state
- Angular momentum (L),
  $L^2 = l(l+1)\left(\frac{h}{2\pi}\right)^2$
- Component of angular momentum in z-direction
  $L_z = \frac{m_l h}{2\pi}$

X-rays

- Cut off wavelength (λ_{min}) of continuous X-ray spectrum is given by
  $\lambda_{min} = \frac{hc}{K_0}$
- Cut off wavelength is totally independent of the target material
- Moseley law
  $\sqrt{\nu} \propto Z$
  or $\sqrt{\nu} = CZ - C$

Nuclei

Atomic Nucleus, its Size and Density

- Nuclear radius, $R = R_0 A^{1/3}$,
  Here, $R_0 = 1.2 \times 10^{-15}$ m
- $\rho_{nu} = \frac{\text{Nuclear mass}}{\text{Nuclear volume}} = \frac{m_{nu}}{\frac{4}{3}\pi R^3} = \frac{3m}{4\pi R_0^3}$
- Average atomic mass of an element = Weighted average of mass of all its isotopes
- 1 amu = 1.66×10^{-27} kg = 931 MeV
- 1 MeV = 1.6×10^{-13} J

Binding Energy

- Mass defect, $\Delta m = [Zm_p + (A - Z)m_n - m_N]$
- Total binding energy = $(\Delta m)c^2$
- Average B.E./nucleon = $\frac{(\Delta m)c^2}{A}$
- Packing fraction = $\frac{M - A}{A}$

Radioactive Decay Law, Half-Life and Average Life

- Disintegration laws for radioactive transformations are as follows
  (a) α-decay $^{A}_{Z}X \longrightarrow\ ^{A-4}_{Z-2}Y + ^{4}_{2}He$
  (b) β-decay $^{A}_{Z}X \longrightarrow\ ^{A}_{Z+1}Y + ^{0}_{-1}e + \bar{\nu}$ (β^- decay)
  $^{A}_{Z}X \longrightarrow\ ^{A}_{Z-1}X + e^+ + \nu$ (β^+ decay)
  (Excited state) (Ground State)
  (c) γ-decay : $^{A}_{Z}X \longrightarrow\ ^{A}_{Z}X + \gamma$
- Radioactive decay law
  (a) $-\frac{dN}{dt} = \lambda N$ (b) $N = N_0 e^{-\lambda t}$
  where, λ = decay constant or disintegration constant
- Half life, $T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$
- $N = N_0\left(\frac{1}{2}\right)^n$, where $n = \frac{t}{T_{1/2}}$
- Mean life, $\tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693} = 1.44T_{1/2}$
  or $T_{1/2} = 0.693\tau$

Activity of a Radioactive Substance

- Decay rate or activity of a substance
  $R = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$
- Time required to reduce the radioactive substances,
  $t = \frac{2.303}{\lambda}\log\frac{N_0}{N}$
- Decay constant, $\lambda = \frac{2.303}{t}\log\frac{N_0}{N}$

Fission and Fusion

- Nuclear fission is splitting of heavy nuclei ($A > 230$) when excited by thermal neutrons, into intermediate mass nuclei,
  e.g. $^{235}_{92}U + ^{1}_{0}n \rightarrow\ ^{236}_{92}U \rightarrow\ ^{140}_{54}Xe + ^{94}_{38}Sr + 2n + Q$
- Nuclear fusion is combining of two light nuclei to form a single large nucleus with energy liberation
  e.g. $^{1}_{1}H + ^{1}_{1}H \rightarrow\ ^{2}_{1}H + e^+ + \nu + 0.42$ MeV

PRACTICE PAPER 2014

BITSAT

Exam from 14th May to 1st June

1. Consider an expression $F = Ax\sin^{-1}(Bt)$ where F represents force, x represents distance and t represents time. Dimensionally the quantity AB represents
   (a) energy (b) surface tension
   (c) intensity of light (d) pressure

2. A juggler throws balls into air. He throws one whenever the previous one is at its highest point. If he throws n balls each second, the height to which each ball will rise is
   (a) $\frac{g}{2n^2}$ (b) $\frac{2g}{n^2}$
   (c) $\frac{2g}{n}$ (d) $\frac{g}{4n^2}$

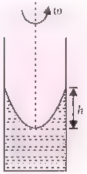
3. Standing waves are produced by the superposition of two waves
   $y_1 = 0.05\sin(3\pi t - 2x)$
   $y_2 = 0.05\sin(3\pi t + 2x)$
   where x and y are in metres and t is in second. What is the amplitude of the particle at $x = 0.5$ m? (Given cos 57.3° = 0.54).
   (a) 2.7 cm (b) 5.4 cm
   (c) 8.1 cm (d) 10.8 cm

4. A vessel has 6 g of hydrogen at pressure P and temperature 500 K. A small hole is made in it so that hydrogen leaks out. How much hydrogen leaks out if the final pressure is $\frac{P}{2}$ and temperature falls to 300 K?
   (a) 2 g (b) 3 g
   (c) 4 g (d) 1 g

5. A particle of mass 0.1 kg is held between two rigid supports by two springs of force constant 8 N m^{-1} and 2 N m^{-1}. If the particle is displaced along the direction of length of the springs, its frequency of vibration is
   (a) $\frac{5}{\pi}$ Hz (b) $\frac{8}{\pi}$ Hz
   (c) $\frac{2}{\pi}$ Hz (d) $\frac{1}{\pi}$ Hz

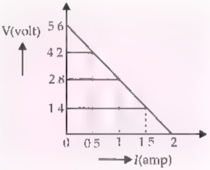
6. A stone is thrown horizontally with velocity u. The velocity of the stone 0.5 s later is $3u/2$. The value of u is
   (a) 2.2 m s^{-1} (b) 3.3 m s^{-1}
   (c) 4.4 m s^{-1} (d) 1.1 m s^{-1}

7. A liquid is kept in a cylindrical vessel which is rotated along its axis. The liquid rises at the sides (see figure). If the radius of the vessel is 0.05 m and the speed of rotation is 2 revolution per second, find the difference in the height of the liquid at the centre of the vessel and its sides



   (a) 20 cm (b) 4 cm
   (c) 2 cm (d) 0.2 cm

8. Four cells of identical emf E and internal resistance r are connected in series to a variable resistor. The following graph shows the variation of terminal voltage of the combination with current. The emf of each cell used is



   (a) 1.4 V (b) 5.6 V
   (c) 2 V (d) 1 V

9. If μ_s is coefficient of static friction, the maximum speed v_{max} with which a vehicle can negotiate an unbanked curved track having radius R and inclined at an angle θ with respect to horizontal plane is

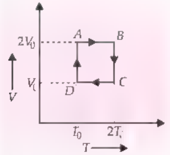
(a) $v_{max} = \sqrt{Rg\tan\theta}$ (b) $v_{max} = \sqrt{\mu_s Rg}$

(c) $\sqrt{Rg}$ (d) $\sqrt{\dfrac{\tan\theta}{Rg}}$

10. A wooden block is dropped from the top of a cliff 100 m high and simultaneously a bullet of mass 10 g is fired from the foot of the cliff upwards with a velocity of 100 m s^{-1}. The bullet and wooden block will meet each other after a time

(a) 10 s (b) 0.5 s
(c) 1 s (d) 7 s

11. One mole of an ideal gas is taken through a cyclic process as shown in the V-T diagram. Which of the following statements are true?



(a) The magnitude of work done by the gas is $RT_0\ln2$.
(b) Work done by gas is V_0T_0.
(c) Net work done by the gas is zero.
(d) Work done by the gas is $2RT_0\ln2$.

12. A telescope has an objective lens of focal length 200 cm and an eye piece with focal length 2 cm. If this telescope is used to see a 50 meter tall building at a distance of 2 km, what is the height of the image of the building formed by the objective lens?

(a) 5 cm (b) 10 cm
(c) 1 cm (d) 2 cm

13. The electric potential between a proton and an electron is given by $V = V_0\ln\left(\dfrac{r}{r_0}\right)$, where r_0 is a constant. Assuming Bohr's model to be applicable, write variation of r_n with n, n being the principal quantum number

(a) $r_n \propto n$ (b) $r_n \propto \dfrac{1}{n}$

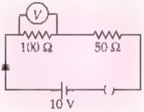
(c) $r_n \propto n^2$ (d) $r_n \propto \dfrac{1}{n^2}$

14. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60°. What is the torque needed to maintain the needle in this position?

(a) $\sqrt{3}W$ (b) W

(c) $\dfrac{\sqrt{3}}{2}W$ (d) $2W$

15. In the given circuit, the voltmeter records 5 V. The resistance of the voltmeter in ohm is



(a) 200 (b) 100
(c) 10 (d) 50

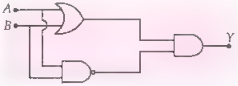
16. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m while applying the force and the ball goes up to 2 m height further, find the magnitude of the force. (Take $g = 10$ m s^{-2})

(a) 16 N (b) 20 N
(c) 22 N (d) 4 N

17. $ABCD$ is a rectangle whose side AB = 10 cm and side BC = 24 cm. A charge of 0.104 μC is lying at the center O of rectangle. If the mid-point of side BC is E, then the work done in carrying 100 μC charge from B to E will be

(a) 1.152 J (b) 2.304 J
(c) 4.082 J (d) 23.4 J

18. Name the gate represented by the following circuit



(a) OR gate (b) XOR gate
(c) AND gate (d) NAND gate

19. Two cylinders of equal size are filled with equal amount of ideal diatomic gas at room temperature. Both the cylinders are fitted with pistons. In cylinder A the piston is free to move, while in cylinder B the piston is fixed. When same amount of heat is supplied to both the cylinders, the temperature of the gas in cylinder A raises by 30 K. What will be the rise in temperature of the gas in cylinder B?

(a) 42 K (b) 30 K
(c) 20 K (d) 56 K

20. A body of mass M at rest explodes into three pieces, two of them of mass $\dfrac{M}{4}$ each, are thrown

off in perpendicular directions with velocities of 3 m s^{-1} and 4 m s^{-1} respectively. The third piece will be thrown off with a velocity of

(a) 1.5 m s^{-1} (b) 2 m s^{-1}
(c) 2.5 m s^{-1} (d) 3 m s^{-1}

21. A variable force, given by the 2-dimensional vector $\vec{F} = (3x^2\hat{i} + 4\hat{j})$, acts on a particle. The force is in newton and x is in metre. What is the change in the kinetic energy of the particle as it moves from the point with coordinates (2, 3) to (3, 0)? (The coordinates are in metres)

(a) −7 J (b) zero
(c) +7 J (d) +19 J

22. A slab of material of dielectric constant K has the same area as the plates of a parallel plate capacitor but has a thickness $\left(\frac{3}{4}\right)d$, where d is the separation of the plates. The ratio of the capacitance C (in the presence of the dielectric) to the capacitance C_0 (in the absence of the dielectric) is

(a) $\frac{3K}{K+4}$ (b) $\frac{3}{4}K$
(c) $\frac{4K}{K+3}$ (d) $\frac{4}{3}K$

23. A pure inductor of 25 mH is connected to a source of 220 V. Given the frequency of the source as 50 Hz, the rms current in the circuit is

(a) 7 A (b) 14 A
(c) 28 A (d) 42 A

24. Light of two different frequencies, whose photons have energies 1 eV and 2.5 eV are respectively, successively illuminate a surface of work function 0.5 eV. The ratio of the maximum speeds of the electrons that are emitted are

(a) 2 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 4

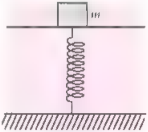
25. Two concentric circular coils of ten turns each are situated in the same plane. Their radii are 20 cm and 40 cm and they carry currents 0.2 A and 0.3 A respectively in opposite directions. The magnetic field in tesla at the centre is

(a) $\frac{30\mu_0}{4}$ (b) $\frac{\mu_0}{80}$
(c) $\frac{7\mu_0}{80}$ (d) $\frac{5\mu_0}{4}$

26. In a two-slit experiment, with monochromatic light, fringes are obtained on a screen placed at some distance from the slits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 10^{-3} m. Then the wavelength of light used is (given that distance between the slits is 0.03 mm)

(a) 4000 Å (b) 4500 Å
(c) 6000 Å (d) 5500 Å

27. A mass of 2.0 kg is put on flat pan attached to a vertical spring fixed on the ground as shown in below figure. The mass of the spring and pan is negligible. When pressed slightly and released, the mass executes a simple harmonic motion. The spring constant is 200 N m^{-1}. What should be the minimum amplitude of the motion so that the mass gets detached from the pan? (Take $g = 10$ m s^{-2})



(a) 10.0 cm
(b) 4.0 cm
(c) 8.0 cm
(d) any value less than 12.0 cm

28. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be

(a) nB (b) n^2B
(c) $2nB$ (d) $2n^2B$

29. Tube A has both ends open while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tubes A and B is

(a) 1 : 2 (b) 1 : 4
(c) 2 : 1 (d) 4 : 1

30. A thin lens of focal length f has a aperture of diameter d. It forms an image of intensity I. Now, the central part of the aperture upto diameter $d/2$ is blocked by opaque paper. The focal length and the image intensity will change to

(a) $\frac{f}{2}, \frac{I}{2}$ (b) $f, \frac{I}{4}$
(c) $\frac{3f}{4}, \frac{I}{2}$ (d) $f, \frac{3I}{4}$

31. A solid cylinder of mass M and radius R rolls without slipping down an inclined plane of length L and height h. What is the speed of its centre of mass when the cylinder reaches its bottom?

(a) $\sqrt{2gh}$ (b) $\sqrt{\frac{3}{4}gh}$
(c) $\sqrt{\frac{4}{3}gh}$ (d) $\sqrt{4gh}$

32. Two simple harmonic motions are represented by
$y_1 = 5[\sin 2\pi t + \sqrt{3}\cos 2\pi t]$ and $y_2 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$
The ratio of their amplitudes is
(a) 1 : 1 (b) 2 : 1
(c) 1 : 3 (d) $\sqrt{3} : 1$

33. Three similar cells, each of emf 2 V and internal resistance r Ω send the same current through an external resistance of 2 Ω, when connected in series or in parallel. The strength of the current flowing through the external resistance is
(a) 1 A (b) 1.5 A
(c) 2 A (d) 0.75 A

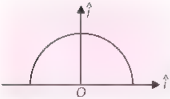
34 In the Bohr's model of hydrogen atom, the ratio of the kinetic energy to the total energy of the electron in n^{th} quantum state is
(a) –1 (b) +1
(c) –2 (d) +2

35 A circular disk of moment of inertia I_t is rotating in a horizontal plane, about its symmetry axis, with a constant angular speed ω_i. Another disk of moment of inertia I_b is dropped coaxially onto the rotating disk. Initially the second disk has zero angular speed. Eventually both the disks rotate with a constant angular speed ω_f. The energy lost by the initially rotating disc to friction is
(a) $\frac{1}{2}\frac{I_b^2}{(I_t+I_b)}\omega_i^2$ (b) $\frac{1}{2}\frac{I_t^2}{(I_t+I_b)}\omega_i^2$
(c) $\frac{I_b-I_t}{(I_t+I_b)}\omega_i^2$ (d) $\frac{1}{2}\frac{I_bI_t}{(I_t+I_b)}\omega_i^2$

36 A cylindrical metallic rod in thermal contact with two reservoirs of heat at its two ends conducts an amount of heat Q in time t. The metallic rod is melted and the material is formed into a rod of half the radius of the original rod. What is the amount of heat conducted by the new rod, when placed in thermal contact with the two reservoirs in time t?
(a) $\frac{Q}{4}$ (b) $\frac{Q}{16}$
(c) $2Q$ (d) $\frac{Q}{2}$

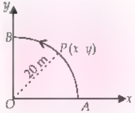
37 A 220 V input is supplied to a transformer. The output circuit draws a current of 2.0 A at 440 V. If the efficiency of the transformer is 80%, the current drawn by the primary windings of the transformer is
(a) 3.6 A (b) 2.8 A
(c) 2.5 A (d) 5.0 A

38. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is



(a) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (b) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$
(c) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (d) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

39 A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of P when $t = 2$ s is nearly



(a) 14 m s^{-2}
(b) 13 m s^{-2}
(c) 12 m s^{-2}
(d) 7.2 m s^{-2}

40. Half-life of a radioactive substance is 20 minute The time between 20% and 80% decay will be
(a) 20 min (b) 30 min
(c) 40 min (d) 25 min

SOLUTIONS

1. **(c)** : Dimensional analysis suggests that the quantity Ax on RHS must have the dimensions of force whereas B must have the dimensions of reciprocal of time. Then, the product AB will have the dimensions of energy per unit area per unit time, the same as those of intensity of light.

2. **(a)** : Time taken by each ball to reach highest point,
$$t = \frac{1}{n}\text{ s}$$
As the juggler throws the second ball, when the first ball is at its highest point, so $v = 0$
Using, $v = u + at$, we get
$$0 = u + (-g)\left(\frac{1}{n}\right)$$
or $u = \left(\frac{g}{n}\right)$ ...(1)
Also $v^2 = u^2 + 2aS$
$$\therefore\ 0 = \left(\frac{g}{n}\right)^2 + 2(-g)h \quad \text{(Using (1))}$$
$$h = \frac{g}{2n^2}$$

3. **(b).** Here, $y_1 = 0.05\sin(3\pi t - 2x)$

$y_2 = 0.05\sin(3\pi t + 2x)$

According to superposition principle, the resultant displacement is

$y = y_1 + y_2 = 0.05\,[\sin(3\pi t - 2x) + \sin(3\pi t + 2x)]$

$y = 0.05 \times 2\sin 3\pi t\cos 2x$

$y = (0.1\cos 2x)\sin 3\pi t = R\sin 3\pi t$

where $R = 0.1\cos 2x$ = amplitude of the resultant standing wave.

At $x = 0.5$m

$R = 0.1\cos 2x = 0.1\cos(2 \times 0.5)$

$= 0.1\cos(1\text{ radian}) = 0.1\cos\dfrac{180^\circ}{\pi}$

$= 0.1\cos 57.3^\circ$

$R = 0.1 \times 0.54\text{ m} = 0.054\text{ m} = 5.4\text{ cm}$

4. **(d):** $PV = \dfrac{m}{M}RT$

Initially, $PV = \dfrac{6}{M}R \times 500$ ...(i)

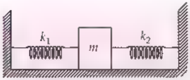
Finally,

$\dfrac{P}{2}V = \dfrac{(6 - x)}{M}R \times 300$ (if x g gas leaks out)

Hence, $2 = \dfrac{6}{6 - x} \times \dfrac{5}{3}$ [Using (i)]

$\therefore\ x = 1$ g

5. **(a):**



Here $m = 0.1$ kg, $k_1 = 8\text{ N m}^{-1}$, $k_2 = 2\text{ N m}^{-1}$

The springs are connected in parallel, the equivalent spring constant is

$k_{eq} = k_1 + k_2 = 8\text{ N m}^{-1} + 2\text{ N m}^{-1} = 10\text{ N m}^{-1}$

The frequency of oscillation is

$$\upsilon = \frac{1}{2\pi}\sqrt{\frac{k_{eq}}{m}} = \frac{1}{2\pi}\sqrt{\frac{(10\text{ N m}^{-1})}{(0.1\text{ kg})}}$$

$$= \frac{10}{2\pi}\text{ Hz} = \frac{5}{\pi}\text{ Hz}$$

6. **(c):** At $t = 0.5$ s, the vertical component of velocity is $v_t = gt = g(0.5) = \dfrac{g}{2} = 4.9\text{ m s}^{-1}$

Hence at $t = 0.5$ s, we have

$$\sqrt{u_t^2 + u^2} = \frac{3u}{2}$$

or $(4.0)^2 + u^2 = \dfrac{9u^2}{4}$

or $5u^2 = 4 \times (4.9)^2$ or $u^2 = (0.8)(4.9)^2$

or $u = (4.9)\sqrt{0.8} = 4.4\text{ m s}^{-1}$

7. **(c):** According to Bernoulli's theorem

$$P + \frac{1}{2}\rho v^2 = \text{ a constant.}$$

Near the ends, the velocity of liquid is higher so the pressure is lower as a result the liquid rises at the sides to compensate for this drop of pressure.

$\rho gh = \dfrac{1}{2}\rho v^2 = \dfrac{1}{2}\rho r^2\omega^2$ $(\because\ v = r\omega)$

$$h = \frac{r^2\omega^2}{2g} = \frac{r^2(2\pi\upsilon)^2}{2g} = \frac{2\pi^2 r^2\upsilon^2}{g}$$

$$= \frac{2 \times \pi^2 \times (0.05)^2 \times 2^2}{9.8} = 0.02\text{ m} = 2\text{ cm}$$

8. **(a):** Emf of each cell $= E$

Internal resistance of each cell $= r$

Number of cells $(n) = 4$

Total emf $= nE = 4E$

Total internal resistance $= nr = 4r$

Terminal voltage,

$V = 4E - I(4r)$

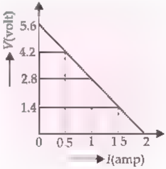
From the graph

When $I = 0$, $V = 5.6$ V

$\therefore$ From (i)

$5.6 = 4E - 0$

$E = \dfrac{5.6}{4} = 1.4$ V



9. **(b).** When a vehicle moves over an unbanked circular track, force of friction provides the necessary centripetal force.

$\therefore\ f = \dfrac{mv^2}{R}$

As $f \le \mu_s N$

$\therefore\ v^2 \le \dfrac{\mu_s RN}{m}$

$v^2 \le \mu_s Rg$ $(\because\ N = mg)$

$\therefore\ V_{max} = \sqrt{\mu_s Rg}$

10. **(c):** Let the bullet and wooden block will meet at time t and at a height x above the foot of the cliff.

Using, $h = ut + \dfrac{1}{2}gt^2$

For the block

$(100 - x) = \dfrac{1}{2}gt^2$ $(\because\ u = 0)$ ...(i)

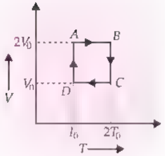
For the bullet

$x = 100t - \dfrac{1}{2}gt^2$ ...(ii)

Adding equations (i) and (ii), we get

$100 = 100t$ or $t = 1$ s

11. (a) :



In the process AB and CD, volume is constant, Therefore, no work is done.

$$W_{DA} = RT_0 \ln\left(\frac{2V_0}{V_0}\right)$$

$$\Rightarrow \quad W_{DA} = RT_0 \ln 2$$

$$W_{BC} = R(2T_0)\ln\left(\frac{V_0}{2V_0}\right) = -2RT_0 \ln 2$$

Total work done by the gas

$W = W_{DA} + W_{BC} = RT_0 \ln 2 - 2RT_0 \ln 2$

$= \quad RT_0 \ln 2$

$|W| = RT_0 \ln 2$

12. (a) : For objective lens, $f_o = +200$ cm, $u_o = -2 \times 10^5$ cm

For a lens

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

or $$\frac{1}{v_o} = \frac{1}{f_o} + \frac{1}{u_o}$$

$$= \frac{1}{200} - \frac{1}{2 \times 10^5} = \frac{999}{2 \times 10^5}$$

or $$v_o = \frac{2 \times 10^5}{999} \text{ cm}$$

Consider magnification due to objective lens

$$\therefore \quad \frac{\text{size of image}}{\text{size of object}} = \left|\frac{v_o}{u_o}\right|$$

$$\therefore \quad \text{Size of image} = \text{size of object} \times \left|\frac{v_o}{u_o}\right|$$

$$= (50 \times 100) \times \frac{2 \times 10^5}{999} \times \frac{1}{2 \times 10^5} = \frac{5000}{999} \text{ cm} \approx 5 \text{ cm}$$

13. (d): According to Bohr's model,

$$r_m = \left(\frac{m^2}{Z}\right) \times r_0$$

where m = orbit number,

r_0 = Bohr radius

For ${}_{100}\text{Fm}^{257}$, $Z = 100 \ \therefore \ m = 5$

$$\therefore \quad r_m = \frac{(5)^2}{100} r_0$$

According to given problem

$$nr_0 = \frac{(5)^2}{100} r_0 \quad \text{or} \quad n = \frac{1}{4}$$

14. (a) : In case of a dipole in a magnetic field,

Workdone, $W = MB(\cos\theta_1 - \cos\theta_2)$

Torque, $\tau = MB\sin\theta$

Here, $\theta_1 = 0$ and $\theta_2 = \theta = 60°$

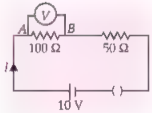
$$\therefore \quad W = MB\,(1 - \cos\theta) = 2MB\sin^2\frac{\theta}{2}$$

$$\tau = MB\sin\theta = 2MB\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$\frac{\tau}{W} = \cot\left(\frac{\theta}{2}\right),$$

or $\tau = W\cot 30° = (\sqrt{3})W$

15. (b):



Let R be the resistance of voltmeter Then effective resistance of circuit is

$$R_{eff} = \frac{R \times 100}{R + 100} + 50 = \frac{150R + 5000}{R + 100}$$

$$\text{Current in the circuit, } I = \frac{10}{(150R + 5000)/(R + 100)}$$

$$= \frac{10(R + 100)}{150R + 5000}$$

Voltage drop across A and B is

$V' = 5 = I \times$ resistance across A and B

or $$5 = \frac{10(R + 100)}{150R + 5000} \times \frac{R \times 100}{(R + 100)} = \frac{1000R}{150R + 5000}$$

On solving, we get $R = 100\ \Omega$

16. (b): Let v be the velocity given by hand and h be the height to which the ball goes.

$$\therefore \quad \frac{1}{2}mv^2 = mgh \qquad \text{...(i)}$$

If F is the force applied by the hand as it moves through 0.2 m, then

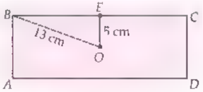
$$\frac{1}{2}mv^2 + F \times 0.2 = mg(h + 2) \qquad \text{...(ii)}$$

Using (i);

$$mgh + F \times 0.2 = mgh + mg \times 2$$

$$F = \frac{mg \times 2}{0.2} = 10mg = 10 \times 0.2 \times 10 = 20 \text{ N.}$$

17. (a) :



$$W = \frac{q_1 q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

$$= 100\times 0.104\times 10^{-12}\times 9\times 10^{9}\left[\frac{1}{0.05}-\frac{1}{0.13}\right]$$

$$= 1.04\times\frac{72}{65} = 1.152\text{ J}$$

18. (b): Output of OR gate is $A+B$. Output of NAND gate is $\overline{A\cdot B}$

Now, $Y = (A+B)\cdot\overline{A\cdot B} = (A+B)\cdot(\bar{A}+\bar{B})$

If $A = 1$ and $B = 1$, then

$A+B = 1$ and $\bar{A}+\bar{B} = 0$

So, $Y = 0$

If $A = 0$ and $B = 0$, then $Y = 0$

If $A = 1$ and $B = 0$, then

$A+B = 1$

$\bar{A}+\bar{B} = 1$

So, $Y = 1$

If $A = 0$ and $B = 1$, then

$Y = 1$

So, the given combination is XOR gate.

19. (a): System A is in isobaric process

$\therefore\ \Delta Q_1 = nC_P\Delta T_1$

System B is in isochoric process

$\Delta Q_2 = nC_V\Delta T_2$

$\because\ \Delta Q_1 = \Delta Q_2$

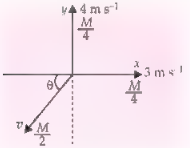
$\therefore\ nC_P\Delta T_1 = nC_V\Delta T_2 \ \therefore\ \Delta T_2 = \frac{C_P}{C_V}\Delta T_1$

For diatomic gas $\frac{C_P}{C_V} = \frac{7}{5}$ and $\Delta T_1 = 30$ K

$\therefore\ \Delta T_2 = \frac{7}{5}\times 30 = 42$ K

20. (c): Mass of third part $= m - \frac{m}{4} - \frac{m}{4} = \frac{m}{2}$

Let us consider that third part of mass $\frac{m}{2}$ will move with velocity v as shown in figure.



Apply linear momentum conservation law along x-axis

$$M\times 0 = \frac{M}{4}\times 3 + \frac{M}{2}(-v\cos\theta)$$

$$\Rightarrow\ v\cos\theta = \frac{3}{2} = v_x \quad \text{...(i)}$$

Similarly along y-axis

$$M\times 0 = \frac{M}{4}\times 4 + \frac{M}{2}(-v\sin\theta)$$

$$\Rightarrow\ v\sin\theta = 2 = v_y \quad \text{(ii)}$$

From (i) and (ii) we get

$$v = \sqrt{v_x^2 + v_y^2}$$

$$v = 2.5\text{ m s}^{-1}$$

21. (c): $\vec{F} = 3x^2\hat{i} + 4\hat{j}$

$\vec{r} = x\hat{i} + y\hat{j}\ \therefore\ d\vec{r} = dx\hat{i} + dy\hat{j}$

Work done, $W = \int \vec{F}\cdot d\vec{r}$

$$= \int_{(2,3)}^{(3,0)} (3x^2\hat{i} + 4\hat{j})\cdot(dx\hat{i} + dy\hat{j})$$

$$= \int_{(2,3)}^{(3,0)} 3x^2dx + 4dy = \int_{(2,3)}^{(3,0)} d(x^3 + 4y)$$

$$= [x^3 + 4y]_{(2,3)}^{(3,0)} = 3^3 + 4\times 0 - (2^3 + 4\times 3)$$

$$= 27 + 0 - (8 + 12) = 27 - 20 = +7\text{ J}$$

According to work energy theorem,

Change in the kinetic energy = Work done, W

$= +7$ J

22. (c): The capacitance of a parallel plate capacitor in the absence of the dielectric is

$$C_0 = \frac{\varepsilon_0 A}{d} \quad \text{...(i)}$$

where A is the area of each plate and d is the distance between them.

The capacitance of a parallel plate capacitor in the presence of dielectric slab of thickness t and dielectric constant K, is

$$C = \frac{\varepsilon_0 A}{(d-t)+\left(\frac{t}{K}\right)} = \frac{\varepsilon_0 A}{\left(d - \frac{3}{4}d\right)+\left(\frac{3d}{4K}\right)}$$

$$C = \frac{\varepsilon_0 A}{\frac{d}{4}+\frac{3d}{4K}} = \frac{4K\varepsilon_0 A}{d(K+3)} \quad \text{...(ii)}$$

Divide (ii) by (i), we get

$$\frac{C}{C_0} = \frac{4K\varepsilon_0 A}{d(K+3)}\times\frac{d}{\varepsilon_0 A} = \frac{4K}{K+3}$$

23. (c): Here, $L = 25$ mH $= 25\times 10^{-3}$ H

$\upsilon = 50$ Hz, $V_{rms} = 220$ V

The inductive reactance is

$$X_L = 2\pi\upsilon L = 2\times\frac{22}{7}\times 50\times 25\times 10^{-3}\ \Omega$$

The rms current in the circuit is

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{220}{2\times\frac{22}{7}\times 50\times 25\times 10^{-3}}$$

$$= \frac{7\times 1000}{2\times 5\times 25}\text{ A} = 28\text{ A}$$

24. **(b):** According to Einstein's photoelectric equation, the maximum kinetic energy of the emitted electron is

$$K_{max} = h\upsilon - \phi_0$$

where,

$h\upsilon$ = photon energy

ϕ_0 = work function

or $\frac{1}{2}mv_{max}^2 = h\upsilon - \phi_0$

where m is the mass of an electron

In the first case

$$\frac{1}{2}mv_{max}^2 = 1\text{ eV} - 0.5 = 0.5\text{ eV} \quad \text{...(i)}$$

In the second case

$$\frac{1}{2}mv_{max}'^2 = 2.5\text{ eV} - 0.5\text{ eV} = 2.0\text{ eV} \quad \text{...(ii)}$$

Divide (i) by (ii) we get

$$\frac{v_{max}^2}{v_{max}'^2} = \frac{0.5}{2.0} = \frac{1}{4} \quad \text{or} \quad \frac{v_{max}}{v'_{max}} = \frac{1}{2}$$

25. **(d):** As the two coils are concentric and in the same plane, carrying currents in opposite directions, the total magnetic field at the centre of the concentric coils is

$$B = B_1 - B_2 = \frac{\mu_0}{4\pi}\frac{2\pi N_1 I_1}{r_1} - \frac{\mu_0}{4\pi}\frac{2\pi N_2 I_2}{r_2}$$

$$= \frac{\mu_0}{2}\left[\frac{N_1 I_1}{r_1} - \frac{N_2 I_2}{r_2}\right]$$

$$= \frac{\mu_0}{2}\left[\frac{10\times0.2}{0.2} - \frac{10\times0.3}{0.4}\right] = \frac{5\mu_0}{4}.$$

26. **(c):** From $\beta = \frac{\lambda D}{d}$ and $\beta' = \frac{\lambda D'}{d}$

$$\beta - \beta' = \frac{\lambda}{d}(D - D'); \; 10^{-3} = \frac{\lambda\times5\times10^{-2}}{0.03\times10^{-3}}$$

$$\lambda = \frac{0.03\times10^{-6}}{5\times10^{-2}} = 6\times10^{-7}\text{ m} = 6000\text{ Å}.$$

27. **(a):** Let x_0 be the compression of the spring when mass m is placed on flat pan. Then

$$mg = kx_0$$

or $x_0 = \frac{mg}{k} = \frac{2\times10}{200} = 0.10\text{ m} = 10\text{ cm}$

If the mass is pressed through a small distance and let it go, it will execute S.H.M. of amplitude a. The mass will get detached from the pan if the restoring force $m\,a\omega^2 > mg$, then mass will move up with same acceleration, detached from the pan, *i.e.*,

$$a > \frac{g}{\omega^2} \quad \text{or} \quad a > \frac{g}{(k/m)}$$

or $a > \frac{10}{(200/2)} > 0.10$ m

The amplitude > 10 cm, *i.e.*, the minimum amplitude is just greater than 10 cm for the given condition. Therefore, the minimum amplitude of oscillation = 10 cm

28. **(b):** The magnetic field at the centre of circular coil is $B = \frac{\mu_0 I}{2r}$

where r = radius of circle $= \frac{l}{2\pi}$ $\quad (\because l = 2\pi r)$

$$\therefore \quad B = \frac{\mu_0 I}{2}\times\frac{2\pi}{l} = \frac{\mu_0 I\pi}{l} \quad \text{...(i)}$$

When wire of length l bents into a circular loop of n turns, then

$$l = n\times2\pi r' \Rightarrow r' = \frac{l}{n\times2\pi}$$

Thus, new magnetic field

$$B' = \frac{\mu_0 nI}{2r'} = \frac{\mu_0 nI}{2}\times\frac{n\times2\pi}{l} = \frac{\mu_0 I\pi}{l}\times n^2 = n^2 B$$

(Using (i))

29. **(c):** Let the tubes A and B have equal length l. Since, tube A is opened at both the ends, therefore, its fundamental frequency

$$\upsilon_A = \frac{v}{2l} \quad \text{...(i)}$$

Since, tube B is closed at one end, therefore, its fundamental frequency

$$\upsilon_B = \frac{v}{4l} \quad \text{...(ii)}$$

From eqs. (i) and (ii), we get

$$\frac{\upsilon_A}{\upsilon_B} = \frac{v/2l}{v/4l} = \frac{4}{2} = 2:1$$

30. **(d):** The focal length of a lens does not change if a part of its blocked. If the central part of the aperture upto $d/2$ is blocked, the exposed area of the aperture reduces by one-fourth the earlier because

$$\frac{\pi\left(\frac{d}{2}\right)^2}{\pi d^2} = \frac{1}{4}$$

Hence the intensity of the image reduces by a factor of 4.

Thus the intensity becomes $I - \frac{I}{4} = \frac{3I}{4}$

31. **(c):** Potential energy of the solid cylinder at height $h = Mgh$.

K.E. of centre of mass when reached at bottom

$$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}MK^2\frac{v^2}{R^2}$$

$$= \frac{1}{2}Mv^2\left(1 + \frac{K^2}{R^2}\right)$$

For a solid cylinder, $\frac{K^2}{R^2} = \frac{1}{2}$

$\therefore$ K.E. $= \frac{3}{4}Mv^2$

$\therefore$ $Mgh = \frac{3}{4}Mv^2$ or $v = \sqrt{\frac{4}{3}gh}$.

32. (b): Here, $y_1 = 5\left[\sin 2\pi t + \sqrt{3}\cos 2\pi t\right]$

$$= 10\left[\frac{1}{2}\sin 2\pi t + \frac{\sqrt{3}}{2}\cos 2\pi t\right]$$

$$= 10\left[\cos\frac{\pi}{3}\sin 2\pi t + \sin\frac{\pi}{3}\cos 2\pi t\right]$$

$$= 10\left[\sin\left(2\pi t + \frac{\pi}{3}\right)\right]$$

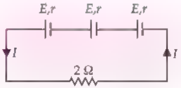
$\therefore$ $A_1 = 10$

and $y_2 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$

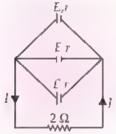
$\therefore$ $A_2 = 5$

Hence $\frac{A_1}{A_2} = \frac{10}{5} = \frac{2}{1}$

33. (d) :



In series,
the effective emf of cells = 3E,
effective internal resistance = 3r
where E is the emf and r is the internal resistance of each cell.



In parallel, the effective emf of cells = E,
effective internal resistance = r/3

As per question, $I = \frac{3E}{2+3r} = \frac{E}{2+\frac{r}{3}}$

or $6 + r = 2 + 3r$ or $r = 2\ \Omega$

$\therefore$ $I = \frac{3\times 2}{2+3\times 2} = \frac{6}{8} = 0.75$ A

34. (a): In Bohr's model of hydrogen atom,
The kinetic energy of the electron in n^{th} state is given by

$$K = \frac{me^4}{8\varepsilon_0^2h^2n^2} = \frac{13.6}{n^2}\text{eV}$$

where $\frac{me^4}{8\varepsilon_0^2h^2} = 13.6$ eV

The potential energy of electron in n^{th} state is given by

$$U = \frac{-2me^4}{8\varepsilon_0^2h^2n^2} = \frac{-27.2}{n^2}\text{ eV}$$

Total energy of electron in n^{th} state is given by

$$E = K + U = \frac{me^4}{8\varepsilon_0^2h^2n^2} - \frac{2me^4}{8\varepsilon_0^2h^2n^2}$$

$$= \frac{-me^4}{8\varepsilon_0 h^2n^2} = \frac{-13.6}{n^2}\text{ eV}$$

$\therefore$ $\frac{K}{E} = -1$

35. (d): As no external torque is applied to the system, the angular momentum of the system remains conserved.

$\because$ $L_i = L_f$

According to given problem,

$I_t\omega_i = (I_t + I_b)\omega_f$

or $\omega_f = \frac{I_t\omega_i}{(I_t + I_b)}$ ...(i)

Initial energy, $E_i = \frac{1}{2}I_t\omega_i^2$ ...(ii)

Final energy, $E_f = \frac{1}{2}(I_t + I_b)\omega_f^2$ (iii)

Substituting the value of ω_f from equation (i) in equation (iii), we get

Final energy, $E_f = \frac{1}{2}(I_t + I_b)\left(\frac{I_t\omega_i}{I_t + I_b}\right)^2$

$$= \frac{1}{2}\frac{I_t^2\omega_i^2}{(I_t + I_b)}$$ ...(iv)

Loss of energy, $\Delta E = E_i - E_f$

$$= \frac{1}{2}I_t\omega_i^2 - \frac{1}{2}\frac{I_t^2\omega_i^2}{(I_t + I_b)}$$ (Using (ii) and (iv))

$$= \frac{\omega_i^2}{2}\left(I_t - \frac{I_t^2}{(I_t + I_b)}\right) = \frac{\omega_i^2}{2}\left(\frac{I_t^2 + I_bI_t - I_t^2}{(I_t + I_b)}\right)$$

$$= \frac{1}{2}\frac{I_bI_t}{(I_t + I_b)}\omega_i^2$$

36. (b): The amount of heat flows in time t through a cylindrical metallic rod of length L and uniform area of cross-section $A(= \pi R^2)$ with its ends maintained at temperatures T_1 and T_2 $(T_1 > T_2)$ is given by

$$Q = \frac{KA(T_1 - T_2)t}{L}$$ ...(i)

where K is the thermal conductivity of the material of the rod.

Area of cross-section of new rod $A' = \pi\left(\frac{R}{2}\right)^2 = \frac{\pi R^2}{4}$

$$= \frac{A}{4}$$ ...(ii)

As the volume of the rod remains unchanged

$\therefore \quad AL = A'L'$

where L' is the length of the new rod.

or $\quad L' = L\dfrac{A}{A'}$ ...(iii)

$= 4L$ (Using (ii))

Now, the amount of heat flows in same time t in the new rod with its ends maintained at the same temperatures T_1 and T_2 is given by

$$Q' = \frac{KA'(T_1 - T_2)t}{L'} \quad ...(iv)$$

Substituting the values of A' and L' from equations (ii) and (iii) in the above equation, we get

$$Q' = \frac{K(A/4)(T_1 - T_2)t}{4L} = \frac{1}{16}\frac{KA(T_1 - T_2)t}{L}$$

$$= \frac{1}{16}Q \quad \text{(Using(i))}$$

37. (d). Here,

Input voltage, $V_p = 220$ V

Output voltage, $V_s = 440$ V

Input current, $I_p = ?$

Output current, $I_s = 2$ A

Efficiency of the transformer, $\eta = 80\%$

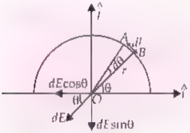
Efficiency of the transformer, $\eta = \dfrac{\text{Output power}}{\text{Input power}}$

$$\eta = \frac{V_s I_s}{V_p I_p}$$

or $\quad I_p = \dfrac{V_s I_s}{\eta V_p} = \dfrac{(440\text{ V})(2\text{ A})}{\left(\dfrac{80}{100}\right)(220\text{ V})}$

$$= \frac{(440\text{ V})(2\text{ A})(100)}{(80)(220\text{ V})} = 5\text{ A}$$

38. (d): Linear charge density, $\lambda = \dfrac{q}{\pi r}$



Consider a small element AB of length dl subtending an angle $d\theta$ at the centre O as shown in the figure.

$\therefore \quad$ Charge on the element, $dq = \lambda dl$

$$= \lambda r d\theta \left(\because \; d\theta = \frac{dl}{r}\right)$$

The electric field at the centre O due to the charge element is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{\lambda r d\theta}{4\pi\varepsilon_0 r^2}$$

Resolve dE into two rectangular components

By symmetry, $\int dE\cos\theta = 0$

The net electric field at O is

$$\vec{E} = \int_0^{\pi} dE\sin\theta(-\hat{j}) = \int_0^{\pi}\frac{\lambda r d\theta}{4\pi\varepsilon_0 r^2}\sin\theta(-\hat{j})$$

$$= -\int_0^{\pi}\frac{qr\sin\theta d\theta}{4\pi^2\varepsilon_0 r^3}\hat{j} \qquad \left(\because \; \lambda = \frac{q}{\pi r}\right)$$

$$= -\int_0^{\pi}\frac{q\sin\theta d\theta}{4\pi^2\varepsilon_0 r^2}\hat{j} = -\frac{q}{4\pi^2\varepsilon_0 r^2}[-\cos\theta]_0^{\pi}\hat{j}$$

$$= -\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$$

39. (a) : Here $s = t^3 + 3$

$\therefore \quad v = \dfrac{ds}{dt} = \dfrac{d}{dt}(t^3 + 3) = 3t^2$

Tangential acceleration, $a_t = \dfrac{dv}{dt} = \dfrac{d}{dt}(3t^2) = 6t$

At $t = 2$ s,

$v = 3(2)^2 = 12$ m s^{-1}, $a_t = 6(2) = 12$ m s^{-2}

Centripetal acceleration,

$$a_c = \frac{v^2}{R} = \frac{(12)^2}{20} = \frac{144}{20} = 7.2\text{ m s}^{-2}$$

Net acceleration,

$$a = \sqrt{(a_c)^2 + (a_t)^2} = \sqrt{(7.2)^2 + (12)^2} \approx 14\text{ m s}^{-2}$$

40. (c) : According to radioactive decay, $N = N_0 e^{-\lambda t}$

where,

N_0 = Number of radioactive nuclei present in the sample at $t = 0$

N = Number of radioactive nuclei left undecayed after time t

λ = decay constant

For 20% decay

$$\frac{80N_0}{100} = N_0 e^{-\lambda t_1} \quad ...(i)$$

For 80% decay

$$\frac{20N_0}{100} = N_0 e^{-\lambda t_2} \quad ...(ii)$$

Dividing equation (i) by (ii), we get

$4 = e^{-\lambda(t_1 - t_2)}$

$\Rightarrow \quad 4 = e^{\lambda(t_2 - t_1)}$

Taking natural logarithms of both sides of above equation, we get

$\ln 4 = \lambda(t_2 - t_1)$

$2\ln 2 = \dfrac{\ln 2}{T_{1/2}}(t_2 - t_1)$

$t_2 - t_1 = 2 \times T_{1/2} = 2 \times 20\text{ min} = 40\text{ min}$

PRACTICE QUESTIONS

TARGET PMTs

Useful for All National and State Level PMTs

1. A projectile has initially the same horizontal velocity as it would acquire if it had moved from rest with uniform acceleration of 3 m s^{-2} for 0.5 minutes. If the maximum height reached by it is 80 m, then the angle of projection is (Take g = 10 m s^{-2})

   (a) $\tan^{-1}(3)$ (b) $\tan^{-1}\left(\frac{3}{2}\right)$

   (c) $\tan^{-1}\left(\frac{4}{9}\right)$ (d) $\sin^{-1}\left(\frac{4}{9}\right)$

2. A horizontal force, just sufficient to move a body of mass 4 kg lying on a rough horizontal surface, is applied on it. The coefficients of static and kinetic friction between the body and the surface are 0.8 and 0.6 respectively. If the force continues to act even after the block has started moving, the acceleration of the block in m s^{-2} is (Take g = 10 m s^{-2})

   (a) 1/4 (b) 1/2

   (c) 2 (d) 4

3. A particle of mass 100 g is thrown vertically upwards with a speed of 5 m s^{-1}. The work done by the force of gravity during the time the particle goes up is

   (a) 0.5 J (b) – 0.5 J

   (c) –1.25 J (d) 1.25 J

4. The moment of inertia of a solid flywheel of radius 0.1 m about its axis is 0.1 kg m^2. A tangential force of 2 kg wt is applied round the circumference of flywheel with the help of string and mass M. The acceleration of the mass is

   (a) 18.6 rad s^{-2} (b) 19.6 rad s^{-2}

   (c) 14.6 rad s^{-2} (d) 16.7 rad s^{-2}

5. A conducting sphere of radius 10 cm is charged with 10 μC. Another uncharged sphere of radius 20 cm is allowed to touch it. After that if the spheres are separated, the surface density of charges on the spheres will be in the ratio of

   (a) 1 : 4 (b) 1 : 2

   (c) 1 : 3 (d) 2 : 1

6. A 100 V voltmeter having an internal resistance of 20 kΩ, when connected in series with a large resistance R across a 110 V line reads 5 V. The magnitude of resistance R is

   (a) 210 kΩ (b) 310 kΩ

   (c) 420 kΩ (d) 440 kΩ

7. A moving coil galvanometer of resistance 100 Ω is used as an ammeter using a resistance 0.1 Ω. The maximum deflection current in the galvanometer is 100 μA. The minimum current in the circuit so that the ammeter shows maximum deflection is

   (a) 100.1 mA (b) 1000.1 mA

   (c) 10.01 mA (d) 1.01 mA

8. The radius of curvature of a thin plano-convex lens is 10 cm (of curved surface) and the refractive index is 1.5. If the plane surface is silvered, then the focal length will be

   (a) 15 cm (b) 20 cm

   (c) 5 cm (d) 10 cm

9. A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27°C. What is the change in the diameter of the hole when the sheet is heated to 227°C? (Take coefficient of linear expansion of copper = 1.70 × 10^{-5} °C^{-1})

   (a) 1.44 × 10^{-2} cm (b) 1.44 × 10^{-3} cm

   (c) 1.44 × 10^{-1} cm (d) 1.44 cm

10. A glass tube of length 1.0 m is completely filled with water. A vibrating tuning fork of frequency 500 Hz is kept over the mouth of the tube and the water is drained out slowly at the bottom of the tube. If velocity of sound in air is 330 m s^{-1}, then the total number of resonances that occur will be

    (a) 2 (b) 3

    (c) 1 (d) 5

11. The efficiency of Carnot engine is 50% and temperature of sink is 500 K. If temperature of source is kept constant and its efficiency raised to 60%, then the required temperature of sink will be
(a) 100 K (b) 600 K
(c) 400 K (d) 500 K

12. At what distance in metre from the centre of the moon, the intensity of gravitational field will be zero? (Take mass of earth and moon as 5.98×10^{24} kg and 7.35×10^{22} kg respectively and the distance between moon and earth as 3.85×10^{8} m.)
(a) zero (b) 3.8×10^{7}
(c) 8×10^{8} (d) 3.7×10^{8}

13. A cylindrical vessel is filled with equal amounts by weights of mercury and water. The total height of the two layers is 29.2 cm. The pressure of the liquid at the bottom of the vessel is (Take specific gravity of mercury =13.6)
(a) 8 cm of Hg (b) 6 cm of Hg
(c) 4 cm of Hg (d) 2 cm of Hg

14. The dimensional formula for acceleration, velocity and length are $\alpha\beta^{-2}$, $\alpha\beta^{-1}$ and $\alpha\gamma$. What is the dimensional formula for the coefficient of friction?
(a) $\alpha\beta\gamma$ (b) $\alpha^{-1}\beta^{0}\gamma^{0}$
(c) $\alpha^{0}\beta^{-1}\gamma^{0}$ (d) $\alpha^{0}\beta^{0}\gamma^{-1}$

15. An ideal choke takes a current of 10 A when connected to an AC supply of 125 V and 50 Hz. A pure resistor under the same conditions takes a current of 12.5 A. If the two are connected to an AC supply of $100\sqrt{2}$ V and 40 Hz, then the current in series combination of above resistor and inductor is
(a) 10 A (b) 12.5 A
(c) 20 A (d) 25 A

16. Maximum velocity of the photo electrons emitted by a metal surface is 1.2×10^{6} m s^{-1}. Assuming the specific charge of the electron to be 1.8×10^{11} C kg^{-1}, the value of the stopping potential in V will be
(a) 2 (b) 3
(c) 4 (d) 6

17. 10^{-3} kg of radioactive isotope of atomic mass 226 emits 3.27×10^{10} α-particles in a second. If 4.2×10^{2} J of energy is released in 1 hour in this process, the average energy of the α-particle is
(a) 1.42 MeV (b) 1.96 MeV
(c) 9.02 MeV (d) 19.6 MeV

18. 10 mA current can pass through a galvanometer of resistance 25 Ω. What resistance in series should be connected through it, so that it is converted into a voltmeter of 100 V?
(a) 0.975 Ω (b) 99.75 Ω
(c) 975 Ω (d) 9975 Ω

19. A 24 kg block resting on a floor has a rope tied to its top. The maximum tension, the rope can withstand without breaking is 310 N. The minimum time in which the block can be lifted a vertical distance of 4.6 m by pulling up the rope is
(a) 1.2 s (b) 1.3 s
(c) 1.7 s (d) 2.3 s

20. A bullet of mass 50 g is fired from a gun of mass 2 kg. If the total kinetic energy produced is 2050 J, the kinetic energy of the bullet and the gun respectively are
(a) 200 J, 5 J (b) 2000 J, 50 J
(c) 5 J, 200 J (d) 50 J, 2000 J

21. Two uniform thin rods each of mass M and length l are placed along X and Y axis with one end of each at the origin. Moment of inertia of the system about Z-axis is
(a) $\frac{3}{2}Ml^{2}$ (b) $\frac{2}{3}Ml^{2}$
(c) $2Ml^{2}$ (d) none of these

22. A mass of 2 kg is put on a flat pan attached to a vertical spring fixed on the ground as shown in the figure. The mass of the spring and the pan is negligible.

When pressed slightly and released the mass executes a simple harmonic motion. The spring constant is 200 N m^{-1}. What should be the minimum amplitude of the motion, so that the mass gets detached from the pan?
(Take g = 10 m s^{-2})
(a) 8 cm
(b) 10 cm
(c) 4 cm
(d) Any value less than 12 cm

23. A solid conducting sphere of radius 10 cm is enclosed by a thin metallic shell of radius 20 cm. A charge q = 20 μC is given to the inner sphere. Find the heat generated in the process when the inner sphere is connected to the shell by a conducting wire.
(a) 24 J (b) 9 J
(c) zero (d) 12 J

24. A conducting circular loop is placed in a uniform magnetic field 0.04 T with its plane perpendicular to the magnetic field. The radius of the loop starts sinking at 2 mm s^{-1}. The induced e.m.f. in the loop when the radius is 2 cm is
(a) $1.6\pi\,\mu V$ (b) $3.2\pi\,\mu V$
(c) $4.8\pi\,\mu V$ (d) $0.8\pi\,\mu V$

25. The image of an electric bulb fixed in a wall is to be obtained on the wall opposite to it at a distance of 3 m. The maximum possible focal length of the convex lens is
(a) 3.25 m (b) 1.55 m
(c) 0.75 m (d) 0.28 m

26. The output Y of the logic circuit shown in figure is best represented as

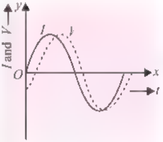
(a) $\overline{A}+B\cdot\overline{C}$ (b) $A+\overline{B}\cdot C$
(c) $\overline{A+B\cdot C}$ (d) $\overline{A+\overline{B}\cdot C}$

27. A ball falls under gravity from a height 10 m with an initial velocity v_0. It hits the ground, loses 50% of its energy in collision and it rises to the same height, what is the value of v_0?
(a) 14 m s^{-1} (b) 7 m s^{-1}
(c) 28 m s^{-1} (d) 9.8 m s^{-1}

28 A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500°C and then placed on a large ice block. What is the maximum amount of ice that can melt? (Take specific heat of copper = 0.39 J g^{-1} K^{-1}, heat of fusion of water = 335 J g^{-1})
(a) 1.5 kg (b) 2.5 kg
(c) 3.5 kg (d) 4.5 kg

29. A string of mass 2.5 kg is under a tension of 200 N. The length of the stretched string is 20 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?
(a) 1 s (b) 0.5 s (c) 1.5 s (d) 2.5 s

30. When an ac source of voltage $V = V_0 \sin 100t$ is connected across a circuit, the phase difference between the voltage V and current I in the circuit is observed to be $\pi/4$, as shown in figure.



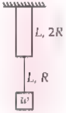
If the circuit consists possibly only of RC or RL or LC in series, find possible values of two elements.
(a) $R = 1$ kΩ, $C = 10$ μF
(b) $R = 1$ kΩ, $C = 1$ μF
(c) $R = 1$ kΩ, $L = 10$ mH
(d) $R = 10$ kΩ, $L = 10$ mH

31. A bullet of mass 0.01 kg and travelling at a speed of 500 m s^{-1} strikes a block of mass 2 kg, which is is suspended by a string of length 5 m. The centre of gravity of the block is found to rise a vertical distance of 0.1 m. What is the speed of the bullet after it emerges from the block?
(a) 580 m s^{-1} (b) 220 m s^{-1}
(c) 1.4 m s^{-1} (d) 7.8 m s^{-1}

32. Two simple pendulums of length 0.5 m and 20 m respectively are given small linear displacement in one direction at the same time. They will again be in the phase when the pendulum of shorter length has completed
(a) 5 oscillations (b) 1 oscillation
(c) 2 oscillations (d) 3 oscillations

33. The frequency of a tuning fork A is 2% more than the frequency of a standard tuning fork. The frequency of another tuning fork B is 3% less than the same standard tuning fork. If 6 beats s^{-1} are heard when the two tuning forks A and B are excited, the frequency of A is
(a) 120 Hz (b) 122.4 Hz
(c) 116.4 Hz (d) 130 Hz

34. Two wires of the same material (Young's modulus Y) and same length L but radii R and $2R$ respectively are joined end to end. A weight w is suspended from the combination as shown in the figure. The elastic potential energy in the system is



(a) $\dfrac{3w^2L}{4\pi R^2Y}$ (b) $\dfrac{3w^2L}{8\pi R^2Y}$
(c) $\dfrac{5w^2L}{8\pi R^2Y}$ (d) $\dfrac{w^2L}{\pi R^2Y}$

35. 22 g of carbon dioxide at 27°C is mixed in a closed container with 16 g of oxygen at 37°C. If both gases are considered as ideal gases, then the temperature of the mixture is
(a) 24.2°C (b) 28.5°C
(c) 31.5°C (d) 33.5°C

36. The electrical conductivity of a semiconductor increases when electromagnetic radiation of wavelength shorter than 2480 nm is incident on it. The band gap for the semiconductor is
(a) 0.5 eV (b) 1.7 eV
(c) 1.1 eV (d) 2.5 eV

37. If the series limit wavelength of the Lyman series for hydrogen atom is 912 Å, then the series limit wavelength of the Balmer series for the hydrogen atom is
(a) 912 Å (b) 912×2 Å
(c) 912×4 Å (d) $\frac{912}{2}$ Å

38. A thin equiconvex lens has focal length 10 cm and refractive index 1.5. One of its faces is now silvered and for an object placed at a distance u in front of the lens, the image coincides with the object. The value of u is
(a) 10 cm (b) 5 cm
(c) 15 cm (d) 20 cm

39. The electric current in a circular coil of two turns produced a magnetic induction of 0.2 T at its centre. The coil is unwound and is rewound into a circular coil of four turns. The magnetic induction at the centre of the coil now is, in tesla (if same current flows in the coil)
(a) 0.2 (b) 0.4
(c) 0.6 (d) 0.8

40. In a series LCR circuit the voltage across the resistance, capacitance and inductance is 10 V each. If the capacitance is short circuited, the voltage across the inductance will be
(a) 10 V (b) $10\sqrt{2}$ V
(c) $(10/\sqrt{2})$ V (d) 20 V

41. A man in a balloon rising vertically with an acceleration of 4.9 m s^{-2} releases a ball 2 s after the balloon is let go from the ground. The greatest height above the ground reached by the ball is (Take g = 9.8 m s^{-2})
(a) 14.7 m (b) 19.6 m
(c) 9.8 m (d) 24.5 m

42. A sand bag of mass M is suspended by rope. A bullet of mass m is fired at it with speed v and gets embeded in it. The loss of kinetic energy is
(a) $\frac{Mmv^2}{2(M+m)}$ (b) $\frac{Mv^2}{2(M+m)}$
(c) $\frac{m^2v^2}{2(M+m)}$ (d) $\frac{1}{2}(M+m)v^2$.

43. A mass of 2.9 kg is suspended from a string of length 50 cm, and is at rest. Another body of mass 100 g, which is moving horizontally with a velocity of 150 m s^{-1} strikes and sticks to it. When the string makes an angle of 60° with the vertical, the tension in the string would be
(a) 270 N (b) 215 N (c) 135 N (d) 95 N

44. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m s^{-1} to the heavier block in the direction of the lighter block. The velocity of the centre of mass is
(a) 30 m s^{-1} (b) 20 m s^{-1}
(c) 10 m s^{-1} (d) 5 m s^{-1}

45. Force on a 1 kg mass on earth of radius R is 10 N. Then the force on a satellite revolving around the earth in the mean orbital radius $3R/2$ will be (Take mass of satellite = 100 kg)
(a) 4.44×10^2 N (b) 3.33×10^2 N
(c) 500 N (d) 6.66×10^2 N

46. A 5 kg brick of dimensions 20 cm × 10 cm × 8 cm is lying on the largest base. It is now made to stand with length vertical. If g = 10 m s^{-2}, then the amount of work done is
(a) 3 J (b) 5 J (c) 7 J (d) 9 J

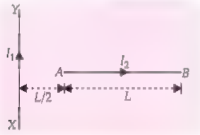
47. A plane electromagnetic wave of frequency 25 MHz travels in free space along the x-direction. At a particular point in space and time, $\vec{E} = 6.3\hat{j}$ V m^{-1}. At this point $\vec{B}$ is equal to
(a) $8.33 \times 10^{-8}\,\hat{k}$ T (b) $18.9 \times 10^{-8}\,\hat{k}$ T
(c) $2.1 \times 10^{-8}\,\hat{k}$ T (d) $2.1 \times 10^{-8}\,\hat{i}$ T

48. For a common emitter amplifier, the audio frequency voltage across the collector resistance 2 kΩ is 2 V. If the current amplification factor of the transistor is 200, and the base resistance is 1.5 kΩ, the input signal voltage and base current are
(a) 0.1 V and 1 μA (b) 0.15 V and 10 μA
(c) 0.015 V and 1 A (d) 0.0075 V and 5 μA

49. Two masses of 40 kg and 30 kg are connected by a weightless string passing over a frictionless pulley as shown in the figure. The tension in the string will be

(a) 168 N (b) 188 N (c) 268 N (d) 368 N

50. A conductor AB of length L carrying a current I_2, is placed perpendicular to a long straight conductor XY carrying current I_1, as shown in figure. The force on AB in magnitude is



(a) $\frac{3\mu_0 I_1 I_2}{2\pi}$ (b) $\frac{\mu_0 I_1 I_2}{2\pi}\ln 2$

(c) $\frac{\mu_0 I_1 I_2}{2\pi}\ln 3$ (d) $\frac{2\mu_0 I_1 I_2}{3\pi}$

SOLUTIONS

1. (c) : As, maximum height $H = \frac{u^2\sin^2\theta}{2g}$

$\therefore \quad 80 = \frac{u^2\sin^2\theta}{2\times 10}$

or $u^2\sin^2\theta = 1600$ or $u\sin\theta = 40$ m s^{-1}

Horizontal velocity $= u\cos\theta$

$= at$

$= 3\times 30 = 90$ m s^{-1}

Now, $\frac{u\sin\theta}{u\cos\theta} = \frac{40}{90}$

or $\tan\theta = \frac{4}{9}$ or $\theta = \tan^{-1}\left(\frac{4}{9}\right)$

2. (c) : The minimum force required to just move a body $= \mu_s N$

The limiting friction, $f_L = \mu_s mg$

The force which is responsible for the increase in velocity of the block,

$F = (\mu_s - \mu_k)mg$

$= (0.8 - 0.6)\times 4\times 10 = 8$ N

$\therefore \quad a = \frac{F}{m} = \frac{8}{4} = 2$ m s^{-2}

3. (c) : For the particle thrown vertically upwards

$m = 100$ g, $u = 5$ m s^{-1}, $v = 0$, $a = g = -10$ m s^{-2}

Let h be the height attained by the particle

$\therefore \quad v^2 - u^2 = 2gh$

or $0 - (5)^2 = 2(-10)\times h$

$\therefore \quad h = 25/20$

Work done by the force of gravity during the time, particle goes up

$$= -mgh = -\frac{100}{1000}\times 10\times\frac{25}{20} = -1.25 \text{ J}$$

4. (d): If T is the tension in the string, then equation of motion of mass M is

$Mg - T = Ma$ ...(i)

where a is the linear acceleration.

If α be the angular acceleration of the flywheel, then

$\tau = I\alpha = TR$ ...(ii)

and $a = R\alpha$ ...(iii)

Putting the value of equations (ii) and (iii) in (i), we have

$$Mg - \frac{I\alpha}{R} = MR\alpha$$

$$\therefore \quad \alpha = \frac{MgR}{I + MR^2} = 16.7 \text{ rad s}^{-2}$$

5. (d): Common potential

$$V = \frac{q_1 + q_2}{C_1 + C_2}$$

$$= \frac{10 + 0}{4\pi\varepsilon_0(0.1 + 0.2)} = \frac{10}{4\pi\varepsilon_0(0.3)}$$

Charges after contact

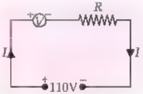
$$q_1' = C_1 V = 4\pi\varepsilon_0(0.1)\times\frac{10}{4\pi\varepsilon_0(0.3)} = \frac{10}{3}$$

$$q_2' = C_2 V = 4\pi\varepsilon_0(0.2)\times\frac{10}{4\pi\varepsilon_0(0.3)} = \frac{20}{3}$$

$$\therefore \quad \frac{\sigma_1'}{\sigma_2'} = \frac{q_1'}{4\pi r_1^2}\times\frac{4\pi r_2^2}{q_2'} = \frac{q_1'}{q_2'}\left(\frac{r_2}{r_1}\right)^2$$

$$= \frac{10}{3}\times\frac{3}{20}\left(\frac{0.2}{0.1}\right)^2 = \frac{1}{2}\times 4 = 2:1.$$

6. (c) :



Current in circuit is

$$I = \frac{110 \text{ V}}{(20 \text{ k}\Omega + R)} \quad \text{...(i)}$$

Also $5 = I\times 20$ k$\Omega = I\times 20\times 10^3$

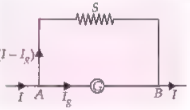
or $\quad I = \frac{5}{20\times 10^3} = \frac{10^{-3}}{4}$ A ...(ii)

From (i) and (ii), we have

$$\frac{10^{-3}}{4} = \frac{110}{20 \text{ k}\Omega + R}$$

On solving, $R = 420$ kΩ

7. (a) : Here, $G = 100\ \Omega$, $S = 0.1\ \Omega$, $I_g = 100\times 10^{-6}$ A,



As, $I_g G = (I - I_g)S = IS - I_g S$

$\therefore \quad I = \dfrac{I_g(G+S)}{S} = \dfrac{(100 \times 10^{-6}) \times (100 + 0.1)}{0.1}$

$= 100.1 \times 10^{-3}$ A = 100.1 mA

8. **(d):** As $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

$= (1.5 - 1)\left(\dfrac{1}{\infty} - \dfrac{1}{-10}\right) = \dfrac{1}{20}$

$f = 20$ cm

When plane surface is silvered,

$\therefore \quad f' = \dfrac{f}{2} = \dfrac{20}{2} = 10$ cm.

9. **(a):** In this problem superficial expansion of copper sheet will be involved on heating.

Here, area of hole at 27°C,

$S_1 = \dfrac{\pi D_1^2}{4} = \dfrac{\pi}{4} \times (4.24)^2$ cm^2

If D_2 cm is the diameter of the hole at 227°C, then area of the hole at 227°C,

$S_2 = \dfrac{\pi D_2^2}{4}$ cm^2

Coefficient of superficial expansion of copper is $\beta = 2\alpha = 2 \times 1.70 \times 10^{-5} = 3.4 \times 10^{-5}$ °C^{-1}

Increase in area $= S_2 - S_1 = \beta S_1 \Delta T$

or $\quad S_2 = S_1 + \beta S_1 \Delta T = S_1(1 + \beta\Delta T)$

$\therefore \quad \dfrac{\pi D_2^2}{4} = \dfrac{\pi}{4}(4.24)^2\,[1 + 3.4 \times 10^{-5} \times (227 - 27)]$

or $\quad D_2^2 = (4.24)^2 \times 1.0068$

$D_2 = 4.2544$ cm

Change in diameter $= D_2 - D_1 = 4.2544 - 4.24$
$= 0.0144$ cm $= 1.44 \times 10^{-2}$ cm

10. **(b):** As $\lambda = \dfrac{v}{\upsilon} = \dfrac{330}{500} = 0.66$ m = 66 cm

This tube is closed at one end. The length of the tube resonating are

$\dfrac{\lambda}{4}, \dfrac{3\lambda}{4}, \dfrac{5\lambda}{4}, \dfrac{7\lambda}{4}$ and so on

$\dfrac{\lambda}{4} = 16.5$ cm, $\dfrac{3\lambda}{4} = 49.5$ cm,

$\dfrac{5\lambda}{4} = 82.5$ cm, $\dfrac{7\lambda}{4} = 115.5$ cm.

But the 7th harmonic needs a length greater than the tube length.

$\because$ There will be three resonances, as the length of the tube is only one meter.

11. **(c):** Efficiency (η) of a Carnot engine is given by

$\eta = 1 - \dfrac{T_2}{T_1}$, where T_1 is the temperature of the source and T_2 is the temperature of the sink.

Here, $T_2 = 500$ K, $\eta = 50\%$

$\therefore \quad 0.5 = 1 - \dfrac{500}{T_1} \Rightarrow T_1 = 1000$ K

Now, $\quad \eta' = 0.6 = 1 - \dfrac{T_2'}{1000}$ (T_2' is the new sink temperature)

$\Rightarrow T_2' = 400$ K.

12. **(b):** Let x be the distance of point from the centre of earth where gravitational intensity is zero. Therefore,

$\dfrac{GM_e}{x^2} = \dfrac{GM_m}{(3.85 \times 10^8 - x)^2}$

or $\quad \dfrac{x}{3.85 \times 10^8 - x} = \sqrt{\dfrac{M_e}{M_m}} = \sqrt{\dfrac{5.98 \times 10^{24}}{7.35 \times 10^{22}}} = 9$

or $\quad \dfrac{x}{9} + x = 3.85 \times 10^8$

or $\quad x = \dfrac{9 \times 3.85 \times 10^8}{10} = 3.47 \times 10^8$ m

$\therefore$ Distance from moon $= 3.85 \times 10^8 - 3.47 \times 10^8$
$= 3.8 \times 10^7$ m

13. **(c):** Let h_1, h_2 be the height of mercury and water layers in the vessel. Let A be the area of cross-section of the vessel. Since their weights are equal so

$(A \times h_1) \times 13.6 \times g = (A \times h_2) \times 1 \times g$

or $\quad h_2 = 13.6\, h_1$

Given, $h_1 + h_2 = 29.2$

$\therefore \quad h_1 + 13.6\, h_1 = 29.2$

or $\quad h_1 = 2$ cm

$\therefore \quad h_2 = 13.6 \times 2 = 27.2$ cm

Total pressure exerted by mercury and water on the bottom of vessel

$= h_1 \times 13.6 \times g + h_2 \times 1 \times g$

$= (2 \times 13.6 \times 980 + 27.2 \times 1 \times 980)$ dyne

$= 2 + \dfrac{27.2}{13.6} = 2 + 2 = 4$ cm of Hg

14. **(d):** Here, $[a] = LT^{-2} = \alpha\beta^{-2}$

$[v] = LT^{-1} = \alpha\beta^{-1}$

$\therefore \quad \alpha = L, \beta = T$

$[L] = \alpha\gamma$

$\therefore \quad \gamma = \dfrac{[L]}{\alpha} = \dfrac{L}{L} = 1$

Coefficient of friction,

$\mu = \dfrac{F}{R} = M^0L^0T^0$ *i.e.* dimensionless

Now, $\alpha^0\beta^0\gamma^{-1} = L^0T^0(1)^{-1} = 1$, which is dimensionless

15. **(a):** $R = \dfrac{V_R}{I_R} = \dfrac{125}{12.5} = 10\ \Omega$

$$X_L = \omega L = 2\pi\upsilon L = \frac{V_L}{I_L} = \frac{125}{10} = 12.5\ \Omega$$

$\therefore\ 2\pi\upsilon L = 12.5\ \Omega$

or $\quad 2\pi L = \frac{12.5}{50} = 0.25$ H

$\therefore\ X_L' = 2\pi L \times \upsilon' = 0.25 \times 40 = 10\ \Omega$

Impedance of the circuit,

$$Z = \sqrt{R^2 + (X_L')^2} = \sqrt{10^2 + 10^2} = 10\sqrt{2}\ \Omega$$

$\therefore$ Current $= \frac{V'}{Z} = \frac{100\sqrt{2}}{10\sqrt{2}} = 10$ A

16. **(c) :** Specific charge of electron,

$$\frac{e}{m} = 1.8 \times 10^{11}\ \mathrm{C\ kg^{-1}}$$

Maximum kinetic energy of photoelectrons

$$= \frac{1}{2}mv_{\max}^2 = eV_s$$

(where V_s is the stopping potential)

$$\therefore\ \frac{1}{2}mv_{\max}^2 = eV_s$$

or $\quad V_s = \frac{mv_{\max}^2}{2e} = \frac{(1.2 \times 10^6)^2}{2 \times 1.8 \times 10^{11}} = 4$ V

17. **(d):** Number of particles emitted per second,

$$\frac{dN}{dt} = \lambda N$$

$\therefore$ Number of particles emitted in 1 hour

$$= \lambda N \times 3600 = \frac{dN}{dt} \times 3600$$

$= 3.72 \times 10^{10} \times 3600$

Energy of all particles $= 4.2 \times 10^2$ J

$\therefore$ Energy of one particle

$$= \frac{4.2 \times 10^2}{3.72 \times 36 \times 10^{12}}\ \mathrm{J} \times \frac{1}{1.6 \times 10^{-13}}\ \mathrm{MeV}$$

$= 19.6$ MeV

18. **(d):** A galvanometer can be converted into a voltmeter of given range by connecting a suitable resistance R in series of galvanometer, which is given by

$$R = \frac{V}{I_g} - G$$

Here, $V = 100$ V, $I_g = 10$ mA $= 10 \times 10^{-3}$ A

$G = 25\ \Omega$,

$$\therefore\ R = \frac{100}{10 \times 10^{-3}} - 25$$

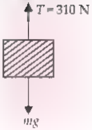
$= 10{,}000 - 25 = 9975\ \Omega$

19. **(c) :** Effective upward force, $F = 310 - mg$

$= 310 - 24 \times 9.8 = 74.8$ N

$\therefore$ Upward acceleration

$$a = \frac{F}{m} = \frac{74.8}{24} = 3.12\ \mathrm{m\ s^{-2}}$$



As $\quad s = ut + \frac{1}{2}at^2$

$\therefore\ 4.6 = 0 + \frac{1}{2} \times 3.12 \times t^2$

or $\quad t^2 = \frac{4.6}{1.56} = 2.95$

or $\quad t = \sqrt{2.95} = 1.7$ s

20. **(b):** Since there is no external force acting on gun bullet system, hence $p_b = p_g$.

$$\because\ K = \frac{p^2}{2m}\ \text{or}\ K \propto \frac{1}{m}$$

$$\therefore\ \frac{K_b}{K_g} = \frac{m_g}{m_b} = \frac{2\ \mathrm{kg}}{50\ \mathrm{g}} = \frac{40}{1}\ \text{or}\ K_g = \frac{K_b}{40}$$

Now total energy,

$$K_b + K_g\ \text{or}\ K_b + \frac{K_b}{40} = \frac{41}{40}K_b = 2050$$

$$\Rightarrow\ K_b = \frac{2050 \times 40}{41} = 2000\ \mathrm{J}$$

and $K_g = 2050 - 2000 = 50$ J.

21. **(b):** According to theorem of parallel axes, moment of inertia of a rod about one of its ends

$$= \frac{Ml^2}{12} + \frac{Ml^2}{4} = \frac{Ml^2}{3}$$

$\therefore$ Moment of inertia of two rods about Z-axis = Moment of inertia of 2 rods placed along X and Y-axis $= \frac{2Ml^2}{3}$

22. **(b):** Let the minimum amplitude of SHM is a. Restoring force on spring, $F = ka$

Restoring force is balanced by weight mg of block. For mass to execute simple harmonic motion of amplitude a, $ka = mg$

or $\quad a = \frac{mg}{k}$

Here, $m = 2$ kg, $k = 200$ N m^{-1}, $g = 10$ m s^{-2}

$$a = \frac{2 \times 10}{200} = \frac{10}{100}\ \mathrm{m} = \frac{10}{100} \times 100\ \mathrm{cm} = 10\ \mathrm{cm}$$

Hence, minimum amplitude of the motion should be 10 cm, so that the mass gets detached from the pan.

23. **(b)**

24. **(b):** $\phi = BA\cos\theta = B(\pi r^2)\cos 0°$

$$|\varepsilon| = \frac{d\phi}{dt} = B\pi(2r)\frac{dr}{dt}$$

$= 0.04 \times \pi(2 \times 2 \times 10^{-2}) \times 2 \times 10^{-3}$
$= 3.2\,\pi \times 10^{-6}$ V $= 3.2\,\pi\,\mu$V

25. **(c) :** For a real image (on wall), the minimum distance between the object and image formed by the convex lens should be four times the focal lens.

$\therefore$ Maximum focal length of the convex lens is
$4f_{max} = 3$ m

$$f_{max} = \frac{3}{4} \text{ m} = 0.75 \text{ m}$$

26. **(d):**

At logic gate *I*, the Boolean expression is

$$\bar{B} \cdot C$$

At logic gate II, the Boolean expression is

$$A + (\bar{B} \cdot C)$$

At logic gate III, the Boolean expression is

$$Y = \overline{A + (\bar{B} \cdot C)}$$

27. **(a) :** Let v be the velocity when it hits the ground. Then from $v^2 = u^2 + 2as$,
$v^2 = v_0^2 + 2g \times 10$
$= v_0^2 + 2 \times 9.8 \times 10$
i.e., $v^2 = v_0^2 + 196$
Let v' be the velocity after impact and it reaches the same height 10 m
$\therefore$ $v'^2 - 0 = 2 \times 9.8 \times 10 = 196 \Rightarrow v' = 14$ m s^{-1}.
Ratio of kinetic energy before impact and after impact

$$\frac{\frac{1}{2}mv^2}{\frac{1}{2}mv'^2} = \frac{v^2}{v'^2} = \frac{v_0^2 + 196}{196} = 2$$

$\therefore$ $v_0^2 = 2 \times 196 - 196 = 196 \Rightarrow v_0 = 14$ m s^{-1}

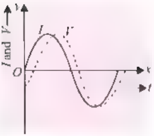
28. **(a) :** Here, mass of copper block, $m = 2.5$ kg $= 2500$ g
Fall in temperature, $\Delta T = 500 - 0 = 500$°C
Specific heat of copper, $s = 0.39$ J g^{-1} K^{-1}
Latent heat of fusion of ice, $L = 335$ J g^{-1}
Let the mass of ice melted be m'.
According to principle of calorimetry,
Heat gained by ice = Heat lost by copper

$$m'L = ms\Delta T$$

$$\therefore\ m' = \frac{ms\Delta T}{L} = \frac{2500 \times 0.39 \times 500}{335} = 1455 \text{ g} \approx 1.5 \text{ kg}$$

29. **(b)**

30. **(a) :** Given figure shows that current *I* leads the voltage V by a phase angle $\pi/4$. Therefore, the circuit can be *RC* circuit alone.



Now, $\tan\phi = \frac{X_C}{R} = \frac{1}{\omega CR}$ $\quad\left(\because X_C = \frac{1}{\omega C}\right)$

$$\tan\frac{\pi}{4} = \frac{1}{\omega CR}$$

$$1 = \frac{1}{\omega CR} \quad \text{...(i)}$$

From $V = V_0 \sin 100t$, we get
$\omega = 100$ rad s^{-1}

$$\therefore\ CR = \frac{1}{\omega} = \frac{1}{100} \quad \text{(Using (i))}$$

When $R = 1$ k$\Omega = 10^3\ \Omega$

$$C = \frac{1}{10^5} = 10^{-5} \text{ F} = 10\ \mu\text{F}$$

31. **(b):** The speed acquired by block, on account of collision of bullet with it, be v_0 m s^{-1}. Since the block rises by 0.1 m, hence

$$0.1 = \frac{v_0^2}{2g}$$

$$\Rightarrow\ v_0^2 = 2 \times g \times 0.1 \text{ or } v_0 = \sqrt{2} \text{ m s}^{-1}$$

Now as per conservation of momentum law for collision between bullet and block,

$$mu = mv + Mv_0$$

$$\Rightarrow\ v = u - \frac{M}{m}v_0 = 500 - \frac{2}{0.01} \times \sqrt{2}$$

$$= (500 - 200\sqrt{2}) \text{ m s}^{-1} \approx 220 \text{ m s}^{-1}$$

32. **(a) :** Let T_1, T_2 be the time period of shorter length and longer length pendulums respectively. As per question,

$$nT_1 = (n-1)T_2,$$

so, $\quad n2\pi\sqrt{\frac{0.5}{g}} = (n-1)2\pi\sqrt{\frac{20}{g}}$

or $\quad n = (n-1)\sqrt{40} \approx (n-1)6$
Hence, $5n = 6$
Hence, after 5 oscillations they will be in same phase.

33. **(b):** Let the frequency of standard fork $= x$

$$\therefore\ \upsilon_A = \frac{102}{100}x,\ \upsilon_B = \frac{97}{100}x$$

Number of beats s^{-1} $= \upsilon_A - \upsilon_B = 6$

or $\quad \frac{102}{100}x - \frac{97}{100}x = 6$

or $x = \frac{6\times100}{5} = 120$ Hz

$\therefore$ Frequency of $A = \frac{102}{100}\times120 = 122.4$ Hz

34. (c)

35. (c) : For carbondioxide, number of moles,

$n_1 = \frac{22}{44} = \frac{1}{2}$

molar specific heat of CO_2 at constant volume, $C_{v_1} = 3R$.

For oxygen, number of moles $n_2 = \frac{16}{32} = \frac{1}{2}$, Molar specific heat of O_2 at constant volume $C_{v_2} = \frac{5R}{2}$.

Let T K be the temperature of mixture.

Heat lost by O_2 = Heat gained by CO_2

$\therefore$ $n_2C_{v_2}\Delta T_2 = n_1C_{v_1}\Delta T_1$

$$\frac{1}{2}\left(\frac{5}{2}R\right)(310-T) = \frac{1}{2}\times(3R)(T-300)$$

or $1550 - 5T = 6T - 1800$

$\therefore$ $T = 304.54$ K $= 31.5°$C

36. (a) : Band gap = Energy of photon of wavelength 2480 nm

$$= \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^8}{2480\times10^{-9}\times1.6\times10^{-19}}\text{eV} = 0.5\text{ eV}$$

37. (c)

38. (b): The radius of curvature of silvered face is given by

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

or $\frac{1}{10} = (1.5-1)\left(\frac{2}{R}\right)$ or $R = 10$ cm

The rays refracted at the unsilvered face will be incident along the radius of the silvered face. So, the virtual image formed by the lens is at the centre of curvature of the silvered face. As $R = 10$ cm so, $v = +10$ cm. Now

$$\frac{1}{v}-\frac{1}{u} = \frac{1}{f} \text{ or } -\frac{1}{10}+\frac{1}{u} = \frac{1}{10}$$

Solving, we get, $u = \frac{10}{2} = 5$ cm

39. (d): When there are two turns in the coil, length of wire $l = 2\times2\pi r_1$

$\therefore$ $r_1 = \frac{l}{4\pi}$

$$B_1 = \frac{\mu_0N_1I}{2r_1} = \frac{2\mu_0I}{l/4\pi} = \frac{2\times4\pi\mu_0I}{l}$$

When there are four turns in the coil,

$l = 4\times2\pi r_2$ $\therefore$ $r_2 = \frac{l}{8\pi}$

$$B_2 = \frac{\mu_0N_2I}{2r_2} = \frac{4\mu_0I}{l/8\pi} = \frac{4\times8\pi\mu_0I}{l}$$

$\therefore$ $\frac{B_1}{B_2} = \frac{8}{32} = \frac{1}{4}$ or $B_2 = 4B_1 = 4\times0.2 = 0.8$ T

40. (c) : Here, $R = X_L = X_C$

($\because$ voltage across them is same)

Total voltage in the circuit,

$V = I[R^2+(X_L-X_C)^2]^{1/2} = IR = 10$ V

When capacitor is short circuited,

$$I = \frac{V}{Z} = \frac{10}{(R^2+X_L^2)^{1/2}} = \frac{10}{\sqrt{2}R}$$

$\therefore$ Potential drop across inductance

$= IX_L = IR = \frac{10}{\sqrt{2}}$ V

41. (a) : For the balloon, $a = 4.9$ m s^{-2}, $t = 2$ s, $u = 0$,

Now, $s = ut + \frac{1}{2}at^2$

$\therefore$ $s = 0+\frac{1}{2}\times4.9(2)^2 = 9.8$ m.

This is the height from where stone is dropped.

Upward velocity of stone,

$v = u + at = 0 + 4.9\times2 = 9.8$ m s^{-1}

The stone will move up till its velocity = 0.

From, $v^2 - u^2 = 2as$

$0-(9.8)^2 = 2(-9.8)s'$ $\therefore$ $s' = 4.9$ m

Maximum height above the ground

$= s + s' = 9.8 + 4.9 = 14.7$ m

42. (a) : According to law of conservation of linear momentum, $mv = (M+m)V$

or $V = \frac{mv}{(M+m)}$

K.E. of the system after collision $= \frac{1}{2}(M+m)V^2$

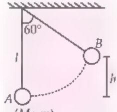
$$= \frac{1}{2}(M+m)\left(\frac{mv}{M+m}\right)^2 = \frac{1}{2}\frac{(mv)^2}{(M+m)}$$

Loss of K.E. $= \frac{1}{2}mv^2 - \frac{1}{2}\frac{(mv)^2}{(M+m)}$

$$= \frac{mv^2}{2}\left[\frac{M+m-m}{M+m}\right] = \frac{mMv^2}{2(M+m)}$$

43. (c) : Here, $M = 2.9$ kg, $l = 50$ cm $= 0.5$ m

$m = 100$ g $= 0.1$ kg, $u = 150$ m s^{-1}, $\theta = 60°$



If v_A is combined velocity at A, then from the principle of conservation of linear momentum,

$$(M+m)v_A = mu$$

$$v_A = \frac{mu}{M+m} = \frac{0.1\times 150}{2.9+0.1} = 5 \text{ m s}^{-1}$$

Height gained in going from A to B,
$h = l(1-\cos\theta) = 0.5(1-\cos 60°) = 0.25$ m
Now, K.E. at B = K.E. at A − $(M+m)gh$

$$\frac{1}{2}(M+m)v_B^2 = \frac{1}{2}(M+m)v_A^2 - (M+m)gh$$

$$v_B^2 = v_A^2 - 2gh = 5^2 - 2\times 9.8\times 0.25 = 20.1 \text{ m}^2\text{ s}^{-2}$$

$$\text{Tension at } B = \frac{(M+m)v_B^2}{l} + (M+m)g\cos\theta$$

$$= \frac{3\times 20.1}{0.5} + 3\times 9.8\times\frac{1}{2} = 135.3 \text{ N}$$

44. **(c)** : Here, $m_1 = 10$ kg, $m_2 = 4$ kg
$v_1 = 14 \text{ m s}^{-1}$, $v_2 = 0$

$$v_{cm} = \frac{10\times 14 + 4\times 0}{10+4} = 10 \text{ m s}^{-1}$$

45. **(a)** : On the surface of earth, the force on a mass of 1 kg is

$$F = \frac{GMm}{R^2} = \frac{GM\times 1}{R^2} = 10 \text{ N}$$

When the radius of the satellite, $r = 3\,R/2$, the force on the satellite is

$$F' = \frac{GMm'}{r^2} = \frac{GM\times 100}{(3/2)^2R^2}$$

$$= \frac{10\times 4\times 100}{9} = 4.44\times 10^2 \text{ N}$$

46. **(a)** :

Initial height of C.G.	= 4 cm
Final height of C.G.	= 10 cm
Increase in height	= 6 cm = 0.06 m
Work done	= 5 × 10 × 0.06 J = 3 J

47. **(c)** : Here, $\vec{E} = 6.3\hat{j}$ V m^{-1}
The magnitude of $\vec{B}$ is

$$B = \frac{E}{c} = \frac{(6.3 \text{ V m}^{-1})}{(3\times 10^8 \text{ m s}^{-1})} = 2.1\times 10^{-8} \text{ T}$$

$\vec{E}$ is along y-direction and the wave propagates along x-axis. Therefore $\vec{B}$ should be in a direction perpendicular to both x and y-axes. Using vector algebra $\vec{E}\times\vec{B}$ should be along x-direction.

Since $(+\hat{j})\times(+\hat{k}) = \hat{i}$, $\vec{B}$ is along z-direction. Thus $\vec{B} = 2.1\times 10^{-8}\,\hat{k}$ T

48. **(d)** : Here,
$V_o = 2$ V, $R_o = 2$ kΩ, $R_i = 1.5$ kΩ, $\beta = 200$

$$A_V = \frac{V_o}{V_i} = \beta\frac{R_o}{R_i}$$

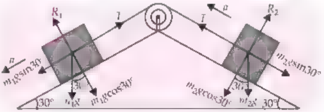
$$\text{or} \quad V_i = \frac{V_oR_i}{\beta R_o} = \frac{2\text{ V}\times 1.5\text{ k}\Omega}{200\times 2\text{ k}\Omega} = 0.0075 \text{ V}$$

$$I_C = \frac{V_o}{R_o} = \frac{2\text{ V}}{2\text{ k}\Omega} = 1 \text{ mA}$$

$$I_B = \frac{I_C}{\beta} = \frac{1\text{ mA}}{200} = 0.005 \text{ mA}$$

$= 5\times 10^{-6}$ A = 5 μA

49. **(a)** : Here, $m_1 = 40$ kg, $m_2 = 30$ kg



Let T be the tension in the string and a is acceleration of the system of two masses.
Their equations of motion are

$m_1a = m_1g\sin 30° - T$ ... (i)

$m_2a = T - m_2g\sin 30°$ ...(ii)

Adding (i) and (ii), we get

$(m_1+m_2)a = (m_1 - m_2)g\sin 30°$

$$(40+30)a = (40-30)9.8\times\frac{1}{2} = 49$$

$$a = \frac{49}{70} = 0.7 \text{ m s}^{-2}$$

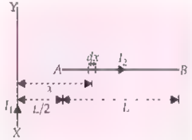
From (ii),

$$T = m_2a + m_2g\sin 30°$$

$$= 30\times 0.7 + 30\times 9.8\times\frac{1}{2}$$

$$= 21 + 147 = 168 \text{ N}$$

50. **(c)** :



Consider an element of length dx on AB at a distance x from XY. Force on the element is

$$dF = \frac{\mu_0}{4\pi}\frac{2I_1}{x}\times I_2\times dx = \frac{\mu_0 I_1 I_2}{2\pi}\frac{dx}{x}$$

Total force on AB is

$$F = \frac{\mu_0}{2\pi}I_1I_2\int_{L/2}^{(3L/2)}\frac{dx}{x} = \frac{\mu_0 I_1 I_2}{2\pi}\ln 3$$

Solved Paper 2014
JEE MAIN

1. When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is
   (a) $\frac{1}{2}\left(\frac{aL^2}{2}+\frac{bL^3}{3}\right)$ (b) $aL^2 + bL^3$
   (c) $\frac{1}{2}\left(aL^2 + bL^3\right)$ (d) $\frac{aL^2}{2}+\frac{bL^3}{3}$

2. The coercivity of a small magnet where the ferromagnet gets demagnetized is 3×10^3 A m^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is
   (a) 6 A (b) 30 mA (c) 60 mA (d) 3 A

3. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be
   (a) 14 A (b) 8 A (c) 10 A (d) 12 A

4. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg)
   (a) 6 cm (b) 16 cm
   (c) 22 cm (d) 38 cm

5. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad s^{-1} about the vertical. About the point of suspension
   (a) angular momentum changes both in direction and magnitude.
   (b) angular momentum is conserved.
   (c) angular momentum changes in magnitude but not in direction.
   (d) angular momentum changes in direction but not in magnitude.

6. The current voltage relation of diode is given by $I = (e^{1000\ V/T} - 1)$ mA, where the applied voltage V is in volts and the temperature T is in degree Kelvin. If a student makes an error measuring $\pm$ 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA?
   (a) 0.05 mA (b) 0.2 mA
   (c) 0.02 mA (d) 0.5 mA

7. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is
   (a) $gH = (n-2)u^2$ (b) $2gH = n^2u^2$
   (c) $gH = (n-2)^2u^2$ (d) $2gH = nu^2(n-2)$

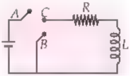
8. A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has the focal lengths f_1 and f_2 respectively. The correct relation between the focal lengths is
   (a) f_1 and f_2 both become negative
   (b) $f_1 = f_2 < f$
   (c) $f_1 > f$ and f_2 becomes negative
   (d) $f_2 > f$ and f_1 becomes negative

9. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is

3×10^4 V/m, the charge density of the positive plate will be close to

(a) 6×10^4 C/m^2 (b) 6×10^{-7} C/m^2
(c) 3×10^{-7} C/m^2 (d) 3×10^4 C/m^2

10. In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant.



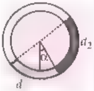
Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to

(a) $\dfrac{1-e}{e}$ (b) $\dfrac{e}{1-e}$
(c) 1 (d) -1

11. Two beams, A and B, of plane polarized light with mutually perpendicular planes of polarization are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation of polaroid through 30° makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then $\dfrac{I_A}{I_B}$ equals

(a) $\dfrac{1}{3}$ (b) 3
(c) $\dfrac{3}{2}$ (d) 1

12. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre.



Radius joining their interface makes an angle α with vertical. Ratio $\dfrac{d_1}{d_2}$ is

(a) $\dfrac{1+\sin\alpha}{1-\cos\alpha}$ (b) $\dfrac{1+\sin\alpha}{1-\sin\alpha}$
(c) $\dfrac{1+\cos\alpha}{1-\cos\alpha}$ (d) $\dfrac{1+\tan\alpha}{1-\tan\alpha}$

13. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is (For steel Young's modulus is 2×10^{11} N m^{-2} and coefficient of thermal expansion is 1.1×10^{-5} K^{-1})

(a) 2.2×10^6 Pa (b) 2.2×10^8 Pa
(c) 2.2×10^9 Pa (d) 2.2×10^7 Pa

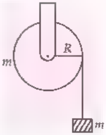
14. A block of mass m is placed on a surface with a vertical cross section given by $y = \dfrac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is

(a) $\dfrac{1}{2}$ m (b) $\dfrac{1}{6}$ m (c) $\dfrac{2}{3}$ m (d) $\dfrac{1}{3}$ m

15. Three rods of Copper, Brass and Steel are welded together to form a Y-shaped structure. Area of cross-section of each rod = 4 cm^2. End of copper rod is maintained at 100° C where as ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cms respectively. The rods are thermally insulated from surroundings except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is

(a) 6.0 cal/s (b) 1.2 cal/s
(c) 2.4 cal/s (d) 4.8 cal/s

16. A mass 'm' is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release ?



(a) g (b) $\dfrac{2g}{3}$ (c) $\dfrac{g}{2}$ (d) $\dfrac{5g}{6}$

17. Match List-I (Electromagnetic wave type) with List-II (Its association/application) and select the correct option from the choices given below the lists :

	List-I		List-II
P	Infrared waves	(i)	To treat muscular strain
Q	Radio waves	(ii)	For broadcasting
R	X-rays	(iii)	To detect fracture of bones
S	Ultraviolet rays	(iv)	Absorbed by the ozone layer of the atmosphere

	P	Q	R	S
(a)	(i)	(ii)	(iii)	(iv)
(b)	(iv)	(iii)	(ii)	(i)
(c)	(i)	(ii)	(iv)	(iii)
(d)	(iii)	(ii)	(i)	(iv)

18. The radiation corresponding to 3 → 2 transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to

(a) 1.6 eV (b) 1.8 eV
(c) 1.1 eV (d) 0.8 eV

19. During the propagation of electromagnetic waves in a medium

(a) Both electric and magnetic energy densities are zero.
(b) Electric energy density is double of the magnetic energy density.
(c) Electric energy density is half of the magnetic energy density.
(d) Electric energy density is equal to the magnetic energy density.

20. A green light is incident from the water to the air-water interface at the critical angle (θ). Select the correct statement.

(a) The entire spectrum of visible light will come out of the water at various angles to the normal.
(b) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal.
(c) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium.
(d) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium.

21. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is :

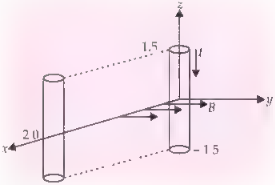
(a) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (b) $\sqrt{\frac{GM}{R}}$

(c) $\sqrt{2\sqrt{2}\frac{GM}{R}}$ (d) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

22. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels $2a$, in same direction, then

(a) time period of oscillations is 6τ
(b) amplitude of motion is $3a$
(c) time period of oscillations is 8τ
(d) amplitude of motion is $4a$

23. A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-\hat{a}_z$ direction (see figure). For a field $\vec{B} = 3.0 \times 10^{-4} e^{-0.2x}\,\hat{a}_y$ T, find the power required to move the conductor at constant speed to $x = 2.0$ m, $y = 0$ m in 5×10^{-3} s. Assume parallel motion along the x-axis.



(a) 29.7 W (b) 1.57 W
(c) 2.97 W (d) 14.85 W

24. The forward biased diode connection is

(a) 

(b) 

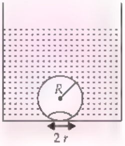
(c) 

(d) 

25. Hydrogen $(_1H^1)$, Deuterium $(_1H^2)$, singly ionised Helium $(_2He^4)^+$ and doubly ionised lithium $(3Li^6)^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wave lengths of emitted radiation are λ_1, λ_2, λ_3 and λ_4 respectively then approximately which one of the following is correct?

(a) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$ (b) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(c) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (d) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

26. On heating water, bubbles being formed at the bottom of the vessel detatch and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If $r << R$, and the surface tension of water is T, value of r just before bubbles detatch is (density of water is ρ_W)



(a) $R^2\sqrt{\frac{3\rho_W g}{T}}$ (b) $R^2\sqrt{\frac{\rho_W g}{3T}}$

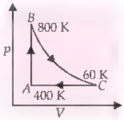
(c) $R^2\sqrt{\frac{\rho_W g}{6T}}$ (d) $R^2\sqrt{\frac{\rho_W g}{T}}$

27. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m s^{-1}.
(a) 4 (b) 12 (c) 8 (d) 6

28. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at $x = 2$ m is
(a) 80 J (b) 120 J (c) −120 J (d) − 80 J

29. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?
(a) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.
(b) A meter scale.
(c) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.
(d) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.

30. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K respectively. Choose the correct statement



(a) The change in internal energy in the process BC is $-500\,R$.
(b) The change in internal energy in whole cyclic process is $250\,R$.
(c) The change in internal energy in the process CA is $700\,R$.
(d) The change in internal energy in the process AB is $-350\,R$.

SOLUTIONS

1. (d): Restoring force, $F = ax + bx^2$
Work done in stretching the rubber-band by a small amount dx is given by
$dW = Fdx$
Net work done in stretching the rubber-band by L is

$$W = \int dW = \int_0^L Fdx$$

$$\Rightarrow W = \int_0^L (ax + bx^2)dx = \left[a\frac{x^2}{2} + b\frac{x^3}{3}\right]_0^L$$

$$\Rightarrow W = \frac{aL^2}{2} + \frac{bL^3}{3}$$

2. (d): Here, $\frac{B}{\mu_0} = 3 \times 10^3$ A m^{-1}
$L = 10$ cm $= 0.1$ m, $N = 100$, $I = ?$

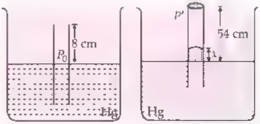
As, $B = \mu_0 nI = \mu_0 \frac{N}{L} I$

$$\Rightarrow I = \frac{B}{\mu_0} \times \frac{L}{N} = 3 \times 10^3 \times \frac{0.1}{100} = 3\text{ A}$$

3. (d): Power of 15 bulbs of 40 W
$= 15 \times 40 = 600$ W
Power of 5 bulbs of 100 W
$= 5 \times 100 = 500$ W
Power of 5 fan of 80 W $= 5 \times 80 = 400$ W
Power of 1 heater of 1 kW $= 1000$ W
∴ Total power, $P = 600 + 500 + 400 + 1000$
$= 2500$ W
When these combination of bulbs, fans and heater are connected to 220 V mains, current in the main fuse of building is given by

$$I = \frac{P}{V} = \frac{2500}{220} = 11.36\text{ A} \approx 12\text{ A}$$

4. (b) :



When glass tube is open, pressure inside it $= P_0$
When the open end of glass tube is closed then pressure inside it $= P'$

$$P' = P_0 - \rho g x \quad \text{...(i)}$$

Work done in case I = Work done in case II
Now, $P_0A(8) = P'A(54 - x)$

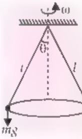
$\Rightarrow P_0(8) = (P_0 - \rho g x)(54 - x)$ [using (i)]

$\Rightarrow \rho g(76)(8) = \rho g(76 - x)(54 - x)$

$\Rightarrow 76(8) = (76 - x)(54 - x)$

On solving $x = 38$ cm
Therefore, air column $= 54 - 38 = 16$ cm

5. (d): $\tau(mg) = mg \times l\sin\theta$
Direction of torque by weight is parallel to the plane of rotation of the particle.



As $\vec{\tau}$ is perpendicular to the angular momentum of the bob so the magnitude of angular momentum remains same but direction changes.

6. (b): As,

$$I = (e^{1000\,V/T} - 1)\text{ mA} \quad \text{...(i)}$$

Here, $I = 5$ mA at $T = 300$ K
$dV = 0.01$ V

$\therefore \quad 5 = (e^{1000\,V/T} - 1)$

$\Rightarrow \quad e^{(1000\,V/T)} = 6$ mA.

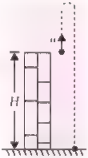
Differentiating eqn. (i), we get

$$dI = \left(\frac{1000}{T}\right) e^{(1000\,V/T)} dV$$

$$= \frac{1000}{300}(6)(0.01) = 0.2 \text{ mA}$$

7. **(d):** Time taken by the particle to reach the top most point is,

$$t = \frac{u}{g} \qquad \text{...(i)}$$



Time taken by the particle to reach the ground $= nt$

Using, $s = ut + \frac{1}{2}at^2$

$$\Rightarrow \quad -H = u(nt) - \frac{1}{2}g(nt)^2$$

$$\Rightarrow \quad -H = u \times n\left(\frac{u}{g}\right) - \frac{1}{2}gn^2\left(\frac{u}{g}\right)^2 \qquad \text{[using (i)]}$$

$$\Rightarrow \quad -2gH = 2nu^2 - n^2u^2 \Rightarrow 2gH = nu^2(n-2)$$

8. **(c) :** Given $\mu = \frac{3}{2}$(crown glass) and focal length $= f$

Focal length $= f_1$ when lens is placed in liquid of refractive index $\mu_1 = \frac{4}{3}$

Focal length $= f_2$ when lens is placed in liquid of refractive index $\mu_2 = \frac{5}{3}$

Using Lens maker's formula

$$\frac{1}{f_1} = \left(\frac{\mu}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{f_1} = \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{8}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Similarly,

$$\frac{1}{f_2} = \left(\frac{\mu}{\mu_2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{f_2} = \left(\frac{3/2}{5/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{-1}{10}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

and $\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Hence, $f_1 = 4f$ and $f_2 = -5f$.

9. **(b):** Here, $K = 2.2$, $E = 3 \times 10^4$ V m^{-1}

Electric field between the parallel plate capacitor with dielectric,

$$E = \frac{\sigma}{K\varepsilon_0}$$

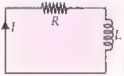
$$\Rightarrow \quad \sigma = K\varepsilon_0 E$$

$$= 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$$

$$= 6 \times 10^{-7} \text{ C m}^{-2}$$

10. **(d):** Initially current in the circuit $= I_0$

After time t current falls to new value.



$$I = I_0\, e^{(-t/\tau)}$$

$\therefore$ Voltage drop across the resistance,

$$V_R = IR = V_0 e^{-t/\tau} \qquad \text{...(i)}$$

Voltage across the inductor,

$$V_L = L\frac{dI}{dt} = L\left[-\frac{I_0}{\tau}e^{(-t/\tau)}\right]$$

$$\Rightarrow \quad V_L = -I_0 R e^{-t/\tau} = -V_0 e^{-t/\tau} \qquad \text{...(ii)}$$

From eqn (i) and eqn (ii)

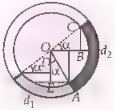
$$\frac{V_R}{V_L} = -1.$$

11. **(a) :** When a polaroid rotated through 30° with respect to beam A, then beam B is at 60° with it.

So, $I_A \cos^2 30° = I_B \cos^2 60°$

$$\Rightarrow \quad I_A\left(\frac{3}{4}\right) = I_B\left(\frac{1}{4}\right) \Rightarrow \frac{I_A}{I_B} = \frac{1}{3}.$$

12. **(d):** $OA = R$



$BC = R\sin\alpha$

$OE = R\cos\alpha$

$OD = R\sin\alpha$

Pressure exerted due to liquid of density d_1 at the point A

$P_1 = P_0 + d_1 g(DE)$

$P_0 + d_1 g(OE - OD)$

$= P_0 + d_1 gR(\cos\alpha - \sin\alpha)$

Pressure exerted due to liquid of density d_2 at the point A

$P_2 = P_0 + d_2 g(AC) = P_0 + d_2 g(BC + OE)$

$= P_0 + d_2 gR\,[\sin\alpha + \cos\alpha)$.

As system is in equilibrium, $P_1 = P_2$.

$$\Rightarrow \quad P_0 + d_1 gR(\cos\alpha - \sin\alpha) = P_0 + d_2 g(\sin\alpha + \cos\alpha)$$

$$\Rightarrow \quad d_1(\cos\alpha - \sin\alpha) = d_2(\sin\alpha + \cos\alpha)$$

$$\Rightarrow \quad \frac{d_1}{d_2} = \frac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha} = \frac{1 + \tan\alpha}{1 - \tan\alpha}.$$

13. **(b):** Given,

$\Delta T = 100°$C, $Y = 2 \times 10^{11}$ N m^{-2}

$\alpha = 1.1 \times 10^{-5}$ K^{-1}

Thermal strain in the wire $= \alpha\Delta T$

[As $l = l_0(1 + \alpha\,\Delta T)$]

Thermal stress in rod is the pressure due to the thermal strain.

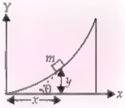
Required pressure $= Y\alpha\Delta T$

$$= 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 100$$

$$= 2.2 \times 10^8 \text{ Pa.}$$

14. (b): Block is under limiting friction, so

$\mu = \tan\theta$ ...(i)



Equation of the surface,

$$y = \frac{x^3}{6}$$

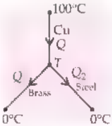
Slope, $\frac{dy}{dx} = \frac{x^2}{2}$ ...(ii)

From eqns (i) and (ii), we get

$$\mu = \frac{x^2}{2} \Rightarrow 0.5 = \frac{x^2}{2} \Rightarrow x^2 = 1 \Rightarrow x = 1$$

So, $y = \frac{1}{6}$.

15. (d): Here, heat flow per second through the copper rod is divided into two parts at the junction and that flow in two different rods made up of brass and steel as shown in figure.



$Q = Q_1 + Q_2$

$$\rightarrow \quad \frac{100 - T}{R_C} = \frac{T - 0}{R_B} + \frac{T - 0}{R_S}$$

where $R = \frac{l}{KA}$, A is equal in each case

$$\rightarrow \quad (100 - T)\frac{K_C}{l_C} = T\left(\frac{K_B}{l_B} + \frac{K_S}{l_S}\right)$$

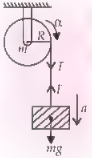
$$\Rightarrow \quad (100 - T)\frac{0.92}{46} = T\left(\frac{0.26}{13} + \frac{0.12}{12}\right)$$

$\Rightarrow \quad T = 40°C$

$$\therefore \quad Q = \frac{(100 - 40)}{l_C} K_C A$$

$$Q = \frac{60 \times 0.92 \times 4}{46} = 4.8 \text{ cal s}^{-1}$$

16. (c): Here, string is not slipping over pulley.



$a = R\alpha$ ...(i)

Applying Newton's second law on hanging block

$mg - T = ma$ ...(ii)

Torque on cylinder due to tension in string about the fixed point

$T \times R = I\alpha$

$T \times R = mR^2\alpha$ $\quad$ ($\because$ $I = mR^2$ for hollow cylinder)

$\Rightarrow \quad T = mR\alpha$

$\Rightarrow \quad T = ma$ $\quad$ [Using eqn. (i)] ...(iii)

From eqns (ii) and (iii)

$$mg = 2ma \quad \Rightarrow \quad a = \frac{g}{2}$$

17. (a): Infrared waves are used to treat muscular strain. Radio waves are used for broadcasting. X-rays are used to defect fracture of bones. Ultraviolet rays are obserbed by the ozone layer of the atmosphere.

18. (c): Radius of a charged particle moving in a constant magnetic field is given by

$$R = \frac{mv}{qB} \text{ or } R^2 = \frac{m^2v^2}{q^2B^2} = \frac{2m\left(\frac{1}{2}mv^2\right)}{q^2B^2} = \frac{2m(K.E.)}{q^2B^2}$$

$$\Rightarrow \quad K.E. = \frac{q^2B^2R^2}{2m}$$

$$\Rightarrow \quad K.E._{max} = \frac{q^2B^2R^2_{max}}{2m} = 0.80 \text{ eV}$$

Energy of photon corresponding transition from orbit $3 \rightarrow 2$ in hydrogen atom.

$$E = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.89 \text{ eV}$$

Using Einstein photoelectric equation.

$E = K.E_{max} + \phi$

$\Rightarrow \quad 1.89 = 0.8 + \phi \quad \Rightarrow \quad \phi = 1.09 \approx 1.1 \text{ eV}$

19. (d): In an em wave, energy is equally divided between the electric and the magnetic fields.

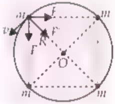
20. (c): As, $\sin\theta = \frac{1}{\mu}$

Also refractive index (μ) of the medium depends on the wavelength of the light. μ is less for the greater wavelength (*i.e.* lesser frequency)

So, θ will be more for lesser frequency of light. Hence, the spectrum of visible light whose frequency is less than that of green light will come out to the air medium.

21. (a): As shown in figure, each mass experiences three forces namely F, F and F'.

Here, F = force between two masses at R separation.



F' = force between two masses at $2R$ separation.

As the all particle move in the circular path of radius R due to their mutual gravitational attraction.

Then net force on mass m

= mass × centripetal acceleration

$$\Rightarrow \frac{F}{\sqrt{2}} + \frac{F}{\sqrt{2}} + F' = m\frac{v^2}{R} \qquad \text{(from figure)}$$

$$\Rightarrow F\sqrt{2} + F' = m\frac{v^2}{R} \Rightarrow \frac{\sqrt{2}Gm^2}{(R\sqrt{2})^2} + \frac{Gm^2}{4R^2} = \frac{mv^2}{R}$$

$$\Rightarrow \frac{Gm}{R}\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right) = v^2 \qquad \Rightarrow v = \frac{1}{2}\sqrt{\frac{Gm}{R}(1 + 2\sqrt{2})}$$

22. **(a)** : As the particle starts from rest so we choose $x = A\cos\omega t$

At $t = 0,\ x = A$

When $t = \tau,\ x = A - a$

When $t = 2\tau,\ x = A - 3a$

$\therefore\quad (A - a) = A\cos\omega\tau \quad ...(i)$

and $(A - 3a) = A\cos 2\omega\tau = A(2\cos^2\omega\tau - 1)$

$$\Rightarrow\quad (A-3a) = A\left[2\left(\frac{A-a}{A}\right)^2 - 1\right]$$

$$\Rightarrow\quad \frac{A-3a}{A} = 2\left(\frac{A-a}{A}\right)^2 - 1$$

On solving, $A = 2a$

Now, $A - a = A\cos\omega\tau$

$$\Rightarrow\quad \cos\omega\tau = \frac{1}{2} = \cos\frac{\pi}{3} \;\Rightarrow\; \omega\tau = \frac{\pi}{3}$$

$$\Rightarrow\quad \frac{2\pi}{T}\tau = \frac{\pi}{3} \Rightarrow T = 6\tau.$$

23. **(c)** : Force on conductor, $\vec{F} = I(\vec{l}\times\vec{B})$

$$\Rightarrow\quad \vec{F} = 10(-3\hat{a}_z)\times(3.0\times10^{-4}e^{-0.2x}\,\hat{a}_y)$$

$$\Rightarrow\quad \vec{F} = 90\times10^{-4}\,(e^{-0.2x}) \text{ along } x\text{-axis}$$

Work done on the conductor in moving along x-axis, $W = \int_{x=0}^{x=2}\vec{F}\cdot\vec{dx}$

$$= 90\times10^{-4}\int_{x=0}^{x=2} e^{-0.2x}dx = 90\times10^{-4}\left[\frac{e^{-0.2x}}{-0.2}\right]_0^2$$

$$\Rightarrow\quad W = 90\times10^{-4}\left[\frac{e^{-0.4}-1}{-0.2}\right]\text{J}$$

This is net work done on the conductor

$$\therefore\quad \text{Average Power, } P_{av} = \frac{\text{Work}}{\text{time}}$$

$$\Rightarrow\quad P_{av} = \frac{90\times10^{-4}(e^{-0.4}-1)}{5\times10^{-3}\times(-0.2)} \Rightarrow P_{av} = 2.97\text{ W}.$$

24. **(b)**:

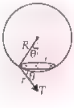
In forward bias, p-side of diode is at higher potential with respect to the potential of n-side.

25. **(d)**: As, $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n^2}\right]$

Here, n_1 and n are same in each case and R is constant

$$\therefore\quad \frac{1}{\lambda} \propto \frac{1}{Z^2} \Rightarrow \lambda \propto Z^2 \;\Rightarrow\; \lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$$

26. **(*)** : Force due to surface tension



$$= \int T\,dl\sin\theta$$

$$= (T\sin\theta)\int dl = T\left(\frac{r}{R}\right)(2\pi r)$$

This force will balance the force of buoyancy.

So, $T(2\pi r)\left(\frac{r}{R}\right) = \rho_W\left(\frac{4}{3}\pi R^3\right)g$

$$\Rightarrow\quad r^2 = \frac{2}{3}\frac{\rho_W g}{T}R^4 \Rightarrow r = R^2\sqrt{\frac{2\rho_W g}{3T}}.$$

* None of given option is correct.

27. **(d)**: Here, $l = 85$ cm $= 0.85$ m

$v = 340$ m s^{-1}

Pipe is closed from one end so it behaves as a closed organ pipe.

Frequencies in the closed organ pipe is given by,

$$\upsilon = \frac{(2n-1)v}{4l} \quad \text{where, } n = 1, 2, 3, 4, \ldots$$

According to question, $\upsilon < 1250$ Hz

$$\left(\frac{2n-1}{4l}\right)v < 1250$$

$$\Rightarrow\quad \frac{(2n-1)\times340}{4\times0.85} < 1250 \;\Rightarrow\; (2n-1) < 12.5$$

Possible value of $n = 1, 2, 3, 4, 5, 6$

So, number of possible natural frequencies lie below 1250 Hz is 6.

28. **(d*)** : Here, $\vec{E} = 30x^2\,\hat{i}$,

V_O is at $x = 0$ and V_A is at $x = 2$ m.

As, $dV = -\vec{E}\cdot d\vec{x}$

$$\text{or}\quad \int_{V_O}^{V_A} dV = -\int_0^2 30x^2dx \quad\Rightarrow [V]_{V_O}^{V_A} = -\left[30\times\frac{x^3}{3}\right]_0^2$$

$$\Rightarrow\quad (V_A - V_O) = -30\times\frac{8}{3} = -80\text{ J C}^{-1}$$

* Given unit in options is wrong.

29. **(c)** : Measured value of the length of rod = 3.50 cm

So, least count of the measuring instrument must be 0.01 cm = 0.1 mm

For, vernier scale, 10 MSD = 1 cm = 10 mm

$\Rightarrow\quad$ 1 MSD = 1 mm

Also, 9 MSD = 10 VSD

L.C. = 1 MSD – 1 VSD

= (1 – 0.9) mm = 0.1 mm

30. **(a)** : Change in internal energy $\Delta U = nC_v\Delta T$

$$= 1\times\frac{5R}{2}\Delta T$$

In the process AB, $\Delta U_{AB} = \frac{5R}{2}(400) = 1000\,R$

In the process BC, $\Delta U_{BC} = \frac{5R}{2}(-200) = -500\,R$

In the process CA, $\Delta U_{CA} = \frac{5R}{2}(-200) = -500\,R$

The change in internal energy in cyclic process is zero.

Thought Provoking Problems

Capacitors

By : Prof. Rajinder Singh Randhawa*

1. A parallel plate capacitor with air as a dielectric is arranged horizontally. The lower plate is fixed and the other is connected with a perpendicular spring. The area of each plate is A. In the steady position, the distance between the plates is d_0 When the capacitor is connected with an electric source with the voltage V, a new equilibrium appears, with the distance between the plates as d_1. Mass of the upper plate is m

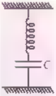


Fig. (1)

   (a) Find the spring constant k.

   (b) What is the maximum voltage for a given k in which an equilibrium can be maintained?

   (c) What is the angular frequency of the oscillating system around the equilibrium value d_1?

2. Three large conducting plates are placed at a distance d apart in air. The space between the first two plates is completely filled with a dielectric slab of dielectric constant $K = 2$ as shown in figure (2).

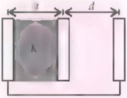
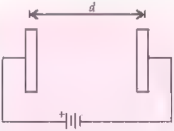


Fig (2)

   The plates are given charges $Q_1 = 7Q$, $Q_2 = 3Q$ and $Q_3 = 2Q$ respectively. The outer two plates are connected with a conducting wire. Find the charges at all surfaces.

3. Consider a parallel plate capacitor whose plate separation is maintained by a dielectric constant K and thickness d when potential across the capacitor is zero.



variable voltage source

Fig (3)

   The dielectric strength of dielectric is E_0 and it is compressible with Young's modulus for compressive stress of Y. The capacitance of capacitor in the limit $V \to 0$ is C_0.

   (a) Derive an expression for the capacitance as a function of voltage across the capacitor.

   (b) What is the maximum voltage that can be applied to the capacitor ?

4. A rectangular parallel plate capacitor of length a and width b has a dielectric of width b partially inserted a distance x between the plates as shown in figure (4).

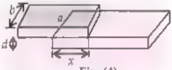


Fig. (4)

   (a) Find the capacitance of the system as a function of x.

   (b) If the capacitor carries charge $+Q$ and $-Q$ on the plates, find the force that acts on the dielectric slab.

   (c) If the capacitor is connected to a constant voltage source V, find the stored energy as a function of x to determine the force that acts on a dielectric slab.

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5. Two concentric spherical shells have radii a and b, the charge on the shells is $+q$ and $-q$ respectively. The space between shells is filled with two dielectrics of constant K_1 and K_2. Each dielectric occupies half the thickness available. Find the capacity of the system between innermost and outermost spheres.

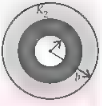


Fig. (5)

SOLUTION

1. (a) Let x_0 be the initial extension in the spring. In equilibrium,

$$kx_0 = mg \quad \text{... (i)}$$

When voltage source is connected the plate separation changes from d_0 to d_1. The extension of the spring increases by $(d_0 - d_1)$.

When equilibrium is attained again, we've

$$k[x_0 + (d_0 - d_1)] = mg + \frac{1}{2}\varepsilon_0 E^2 A \text{ where } E = \frac{V}{d_1}$$

Solving for k, $k = \dfrac{\varepsilon_0 A V^2}{2d_1^2(d_0 - d_1)}$ ... (ii)

(b) From equation. (ii),

$$V^2 = \frac{2kd_1^2(d_0 - d_1)}{\varepsilon_0 A}$$

Differentiating with respect to d_1, we get

$$2V\left(\frac{dV}{dd_1}\right) = \frac{2k}{\varepsilon_0 A}[2d_1 d_0 - 3d_1^2]$$

For V to be maximum, $\left(\dfrac{dV}{dd_1}\right) = 0$

On solving for d_1, we get

$$d_1 = \left(\frac{2}{3}d_0\right)$$

Hence, $V_{max}^2 = \dfrac{2k\left(\frac{2}{3}d_0\right)^2\left[d_0 - \frac{2}{3}d_0\right]}{\varepsilon_0 A}$

$$\Rightarrow V_{max} = \sqrt{\frac{k}{\varepsilon_0 A}}\left(\frac{2}{3}d_0\right)^{3/2} \quad \text{(iii)}$$

(c) Let the small displacement of the upper plate be x downwards from equilibrium position. Then the net force on the plate is

$$F = -k[x_0 + (d_0 - d_1) + x] + mg + \frac{1}{2}\varepsilon_0 A\left[\frac{V^2}{(d_1 - x)^2}\right]$$

$$F = -k[d_0 - d_1] - kx + \frac{1}{2}\varepsilon_0 \frac{AV^2\left(1 + \frac{2x}{d_1}\right)}{d_1^2}$$

[Using binomial expansion]

So, $F = -k(d_0 - d_1) - kx + k(d_0 - d_1)\left(1 + \dfrac{2x}{d_1}\right)$

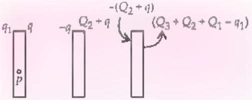
$$= -kx\left[\frac{3d_1 - 2d_0}{d_1}\right] \quad \text{(Using (ii))}$$

Acceleration, $a = \dfrac{F}{m} = \dfrac{-k}{m}\left[\dfrac{3d_1 - 2d_0}{d_1}\right]x$ ... (iv)

Compare it with $a = -\omega^2 x$, we get

$$\omega = \sqrt{\frac{k}{m}\left(\frac{3d_1 - 2d_0}{d_1}\right)}$$

2. Let the distribution of charges be as shown in figure .



As electric field inside the conductor (let at point P) is zero,

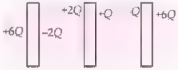
$$q_1 = Q_3 + Q_2 + Q_1 - q_1 \text{ or } q_1 = \frac{Q_3 + Q_2 + Q_1}{2}$$

As the outer plates are connected, their potential will be same.

$$\frac{Q_2 + q}{\varepsilon_0 A}d + \frac{q}{K\varepsilon_0 A}d = 0$$

$$\therefore Q_2 + q + \frac{q}{K} = 0 \Rightarrow q = -\frac{KQ_2}{K+1} = -2Q$$

Thus the final charge configuration on six surfaces will be as shown.



3. (a) Let the voltage applied and compression in the dielectric be V and x respectively. Now we consider small compresion dx during that charge supplied by battery is dq

From conservation of energy,

work done by battery = electrostatic energy in dielectric + potential energy in dielectric

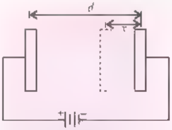


Fig. (3)

$$\therefore \quad Vdq - \frac{1}{2}Vdq + \frac{YA}{d}x\,dx$$

$$\Rightarrow \frac{1}{2}Vdq = \frac{YA}{d}x\,dx \qquad \text{... (i)}$$

As $q = KCV$, $dq = K(dC)V$ ... (ii)

Put in (i), $\frac{1}{2}KV^2dC = \frac{YA}{d}xdx$ ... (iii)

Capacity of the capacitor is $C = \frac{K\varepsilon_0 A}{d-x}$ ... (iv)

$$\therefore \quad \frac{dC}{dx} = \frac{K\varepsilon_0 A}{(d-x)^2} \qquad \text{... (v)}$$

Now, put (v) in (iii), we get

$$\frac{1}{2}KV^2\frac{K\varepsilon_0 A}{(d-x)^2} = \frac{YA}{d}x \Rightarrow V^2 = \frac{2Y(d-x)^2}{K^2\varepsilon_0 d}x \quad \text{...(vi)}$$

For a small compression x, *i.e.* $x << d$,

$d - x \approx d$

$$\therefore \quad V^2 = \frac{2Yd^2}{K^2\varepsilon_0 d}x \Rightarrow x = \frac{K^2\varepsilon_0 V^2}{2Yd} \qquad \text{... (vii)}$$

Put (vii) in (iv), we get

$$C = \frac{K\varepsilon_0 A}{d - \frac{V^2K^2\varepsilon_0}{2Yd}} = \frac{K\varepsilon_0 A}{d\left[1 - \frac{V^2K^2\varepsilon_0}{2Yd^2}\right]}$$

$$\text{If } x << d,\ C = \frac{K\varepsilon_0 A}{d}\left[1 - \frac{V^2K^2\varepsilon_0}{2Yd^2}\right]^{-1}$$

$$C = \frac{K\varepsilon_0 A}{d}\left[1 + \frac{V^2K^2\varepsilon_0}{2Yd^2}\right]. \text{ As } V = 0,\ C_0 = \frac{K\varepsilon_0 A}{d}$$

$$\therefore \quad C = C_0\left[1 + \frac{V^2K^2\varepsilon_0}{2Yd^2}\right]$$

(b) Let x_0 be the compression when dieletric breakdown occurs. The potential difference at this position, $V = E_0(d - x_0)$.

Put it in equation (vi), we get

$$E_0^2 = \frac{2Y}{K^2\varepsilon_0 d}x_0 \Rightarrow x_0 = \frac{E_0^2K^2\varepsilon_0 d}{2Y}$$

$$\therefore \quad V_0 = E_0\left[d - \frac{E_0^2K^2\varepsilon_0 d}{2Y}\right] = E_0 d\left[1 - \frac{E_0^2K^2\varepsilon_0}{2Y}\right]$$

4. (a) : The system is a parallel combination of two capacitors, one with empty space and other with dielectric slab. The equivalent capacity is

$C_{eq} = C_{dielectric} + C_{empty}$

$$= \frac{K\varepsilon_0 b(x)}{d} + \frac{\varepsilon_0 b(a-x)}{d} = \frac{\varepsilon_0 b}{d}[a + (K-1)x] \quad \text{...(i)}$$

(b) The energy of the capacitor

$$U = \frac{Q^2}{2C} = \frac{Q^2 d}{2\varepsilon_0 b}\frac{1}{[a + (K-1)x]} \qquad \text{(ii)}$$

In the absence of any source charge of the capacitor remains constant. As electrostatic field is conservative, we have

$$F = -\frac{dU}{dx} = \frac{-Q^2 d}{2\varepsilon_0 b}\frac{d}{dx}\left[\frac{1}{a + (K-1)x}\right]$$

$$= \frac{Q^2 d(K-1)}{2\varepsilon_0 b[a + (K-1)x]^2} \qquad \text{... (iii)}$$

Using equation (i), we get

$$F = \frac{1}{2}\frac{\varepsilon_0 b}{d}\left(\frac{Q}{C_{eq}}\right)^2(K-1)$$

(c) When the constant voltage source is attached, then

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_0 b}{d}[a + (K-1)x]V^2$$

Since $F = -\frac{dU}{dx}$

$$\therefore \quad F = \frac{1}{2}\frac{\varepsilon_0 b}{d}V^2\frac{d}{dx}[a + (K-1)x]$$

$$F = \frac{1}{2}\frac{\varepsilon_0 b}{d}(K-1)V^2$$

5. Inner capacitor has radii, $r_1 = a$

$$r_2 = a + \left(\frac{b-a}{2}\right) = \frac{a+b}{2}$$

Hence,

$$C_1 = \frac{4\pi\varepsilon_0 K_1 a\left[\frac{a+b}{2}\right]}{\left[\frac{a+b}{2}\right] - a} = \frac{4\pi\varepsilon_0 K_1 a(a+b)}{(b-a)}$$

Outer capacitor has radii, $r_1 = \frac{a+b}{2}$, $r_2 = b$

Hence,

$$C_2 = \frac{4\pi\varepsilon_0 K_2 b\left[\frac{a+b}{2}\right]}{b - \left[\frac{a+b}{2}\right]} = \frac{4\pi\varepsilon_0 K_2 b(a+b)}{(b-a)}$$

The equivalent capacity of the system is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{(b-a)}{4\pi\varepsilon_0 K_1 a(a+b)} + \frac{(b-a)}{4\pi\varepsilon_0 K_2 b(a+b)}$$

On solving for C_{eq}, we get

$$C_{eq} = 4\pi\varepsilon_0 K_1 K_2 ab\frac{(a+b)}{(b-a)}\frac{1}{[K_1 a + K_2 b]}$$

PHYSICS MUSING

SOLUTION SET-9

1. **(b,c)** : Image can be formed on the screen if it is real. Real image of reduced size can be formed by placing object beyond centre of curvature before a concave mirror or a convex lens.
Let $u = -(2f + x)$, then

For convex lens, $-\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

or $\quad \frac{1}{2f + x} + \frac{1}{v} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{2f+x} = \frac{f+x}{f(2f+x)}$$

$$\therefore \quad v = \frac{f(2f+x)}{f+x}$$

It is given that $|u| + |v| = 1$ m

$$2f + x + \frac{f(2f+x)}{f+x} = (2f+x)\left[1 + \frac{f}{f+x}\right] = 1$$

or $\quad \frac{(2f+x)^2}{f+x} = 1 \quad$ ...(i)

Also, for diminished image, $m < 1$, which gives $2f + x < 1$

$(f + x) < 1 \quad$ ...(ii)

This will be true only when $f < 0.25$ m.

2. **(b,c)** : Equivalent focal length of two lenses separated by distance x, $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$

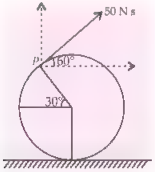
$\therefore \quad \frac{1}{f} = \frac{1}{f_1} - \frac{1}{f_1} + \frac{x}{f_1^2}$ ($\because$ both lenses have same power)

or $\quad \frac{1}{f} = \frac{x}{f_1^2} \Rightarrow f > 0$ for every x.

For $x = 0$, $f = \infty$

Hence for $x = 0$, system will behave like a glass plate.

3. **(c)** :



Impulse = change in momentum, $P = 50$ N s

Velocity along vertical direction

$$v_V = \frac{P\sin 60^\circ}{m} = \frac{50\sin 60^\circ}{1}\text{ m s}^{-1} = 25\sqrt{3}\text{ m s}^{-1}$$

Velocity along horizontal direction,

$$v_H = \frac{P\cos 60^\circ}{m} = \frac{50\cos 60^\circ}{1}\text{ m s}^{-1} = 25\text{ m s}^{-1}$$

$\because \quad$ Time of flight, $T = \frac{2v_V}{g} = 5\sqrt{3}$ s

Also, $\omega = \frac{L}{I} = \frac{P \times r}{\frac{2}{5}mr^2} = \frac{50 \times 0.2 \times 5}{2 \times 1 \times (0.2)^2}$

$= 625$ rad s^{-1}

As $\quad 2\pi n = \omega T$

$$\therefore \quad n = \frac{\omega T}{2\pi} = \frac{625 \times 5\sqrt{3}}{2\pi} = \frac{3125\sqrt{3}}{2\pi}$$

4. **(d)**: Frictional force between ground and sphere,

$$f = \frac{F}{1 + \frac{mr^2}{I_{cm}}}$$

For no slipping $f \le \mu N$

$$\Rightarrow \quad \frac{F}{\left(1 + \frac{mr^2}{I_{cm}}\right)} < \mu mg$$

As, $I_{cm} = \frac{2}{5}mr^2$

$\therefore \quad \frac{2F}{7} \le \mu mg \quad \rightarrow \quad F \le \frac{7\mu mg}{2}$

$\therefore \quad$ The maximum value of $F = \frac{7}{2}\mu mg$

5. **(c)** : Angular acceleration, $\alpha = \frac{-\mu mgr}{\frac{mr^2}{2}} = \frac{-2\mu g}{r}$

Time taken to stop rotation, $t_1 = \frac{\omega_0}{|\alpha|} = \frac{\omega_0 r}{2\mu g}$

Linear acceleration, $a = -\mu g$

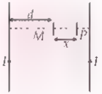
Time taken to stop translation, $t_2 = \frac{v_0}{|a|} = \frac{\omega_0 r}{4\mu g}$

$\because \quad t_2 < t_1$

Hence disc will return to initial point.

6. **(b)**: The magnetic field due to two wires at P

$$B_1 = \frac{\mu_0 I}{2\pi(d + x)}$$



$$B_2 = \frac{\mu_0 I}{2\pi(d - x)}$$

Both the magnetic fields act in opposite direction.

$$\therefore \quad B = B_2 - B_1 = \frac{\mu_0 I}{2\pi}\left[\frac{1}{d-x} - \frac{1}{d+x}\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{d + x - d + x}{d^2 - x^2}\right] = \frac{\mu_0 I x}{\pi(d^2 - x^2)}$$

Contd. on page no. 87

PROBLEM SOLVING TACTICS

OPTICAL IMAGE FORMATION AT OBJECT ITSELF

*By : Sameer Gupta

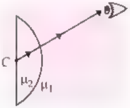
Dealing problems in optics, when optical image is formed by the systems/system of optical elements, on the object itself. These situations can be result of generally retracing of the path of the light and other due to non bending of the light while travelling through the optical media.

How these problems are to be tackled with, here are some of the optical concepts to deal with such type of problems.

1. In the case of refraction through the single curved surface, the situation can be little interesting. If the light is refracting through the single refracting surface from the denser to the rarer medium, the optical formula is

   $\frac{\mu_2}{-u}+\frac{\mu_1}{v}=\frac{(\mu_1-\mu_2)}{R}$ leads to $\frac{\mu_2}{(-u)}+\frac{\mu_1}{(-u)}=\frac{(\mu_1-\mu_2)}{(-R)}$

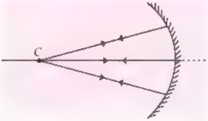
   Solution for this is when $u = R$. Thus the object and the image coincide when the object in this case is kept at the centre of curvature.



   For a transparent lens, which is not polished from any of the side, such attempt to make the image on itself is not possible as $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ will give $\frac{1}{-u}-\frac{1}{-u}=\frac{1}{f}$

   which is not possible as an optical lens do have some power of convergence or divergene.

2. In the case of concave mirror, as the light here can retrace the path itself and $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ will give $\frac{1}{-u}+\frac{1}{-u}=\frac{1}{-R/2}$ which leads to $u = R$.



Thus, it is clear from both the points 1 and 2 that for the single refracting surfaces and for the concave mirrors, the object and the image will coincide with each other if $u = R$

But here along with it, it is interesting to note that, in the case of diverging mirror the situation might seems to be same, but how can one place a real object on the centre of curvature of such mirror? Thus the situation is much valid for the virtual objects. Or the convex mirror may work with other optical element to make the light get back through the same path.

Retracing from the other optical elements can also be done, with the help of mirror used at the end of the various transparent optical elements train. Here different types of mirrors can play a different role for the image formation on object itself.

1. A plane mirror only retraces the light when the light is incident normally on the plane mirror. Hence it is clear that in these type of optical situations the light coming out from the last transparent optical element should come out in such a way that it simply strikes normally the plane mirror surface.

2. In the optical situation where, the last optical element is a convex mirror, the light from the

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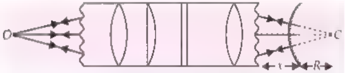
various transparent elements, when oozes out from the system, should be targeted convergence along the centre of curvature R of the convex mirror. If the distance of the last transparent element from this mirror is x, hence the last transparent element should be beamed at $x + R$.

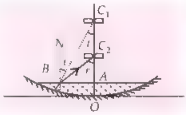


3. For the light to be retraced from the surface of the concave mirror, at the last of the optical elements, the beam is first made to converge at the centre of curvature, before to strike at the surface of concave mirror. If the distance of the last transparent element from this mirror is x, hence the last transparent element should be beamed at $x - R$.

Here are some of the experimental problems for such a situation, to understand the concept in a better way.

1. Measurement of refractive index of a liquid using concave mirror

Place concave mirror of large focal length on the table with mirror surface upward and vertical principal axis. With the help of a sliding smooth stand a horizontal pin is placed with the tip of the pin on the principal axis of the mirror. Pin is to be slided till there is no parallax between the tip of the pin and the image. The pin here is on the centre of curvature of the mirror.



Now add small quantity of water/liquid whose refractive index is to be calculated. Move the pin down till there is again no parallax between the pin and the image. If C_1 is the parallax position without liquid and C_2 is the position with the liquid on the mirror, then $\mu = OC_1/OC_2$

In ΔC_1AB, $\sin i = \dfrac{AB}{BC_1}$

and in ΔC_2AB, $\sin r = \dfrac{AB}{BC_2}$

As from Snell's law, $\mu \sin i = 1 \sin r$

$$\Rightarrow \quad \mu = \frac{\sin r}{\sin i} = \frac{AB}{BC_2} \times \frac{BC_1}{AB} \quad \therefore \quad \mu = \frac{BC_1}{BC_2}$$

As for paraxial rays, $\mu = \dfrac{AC_1}{AC_2}$

If thickness of liquid can be neglected then $\mu = \dfrac{OC_1}{OC_2}$

2 Measurement of the focal length of a convex lens and refractive index of a liquid using convex lens: Keep the convex lens on a plane mirror. As stated in the earlier experiment 1, place the pin where the pin gives no parallax with the image. Here the pin is on the principal focus of the lens. Further one can calculate the refractive index of the material of the lens also. By a calculating method one can easily see that, one object is at the focus of the lens, the refracted rays from the lens, emerge as parallel rays and strike normally on the plane mirror to retrace the path and again to get its focus on the same side

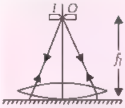


Fig. I

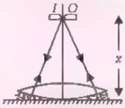


Fig. II

Here in this method, a transparent liquid's refractive index can also be calculated. Add few drops of the liquid between the lens and the mirror and repeat the experiment to get the parallax position again. If R is the radius of curvature of the equi-convex lens and x is the position of the new parallax, then $\mu = 1 + \dfrac{R}{x}$

From fig. I

$$\frac{1}{F} = \frac{1}{f_l} + \frac{1}{f_m} + \frac{1}{f_l}$$

$$\frac{1}{F} = \frac{2}{f_l}$$

$2F = f_l$ system will behave like concave mirror
$x \;\; f_l$

From fig. II

$$\frac{1}{F} = \frac{1}{f_l} + \frac{1}{f_w} + \frac{1}{f_m} + \frac{1}{f_w} + \frac{1}{f_l}$$

$$\frac{1}{F} = \frac{2}{f_l} + \frac{2}{f_w} + 0$$

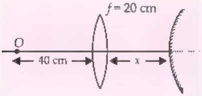
$$\frac{1}{F} = 2\left[\frac{(\mu-1)2}{R}\right] + 2\left[-\frac{(\mu-1)1}{R}\right]$$

$$\frac{1}{F} = \frac{2(\mu-1)}{R} \Rightarrow 2F = \frac{R}{(\mu-1)}$$

Hence $x = \dfrac{R}{(\mu-1)}$, $\mu = 1 + \dfrac{R}{x}$

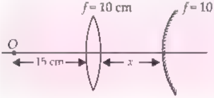
TEST YOURSELF

1 A point object O is placed at distance of 40 cm from a convex lens of focal length 20 cm as shown in the figure. At what distance x from the lens should a convex mirror of focal length 50 cm be placed, so that final image coincide with the object?



[Ans : 40 cm]

2. In the given figure, what should be the value of x so that image is formed on the object itself?



[Ans : 10 cm]

3. A thin bi-convex lens of refractive index 3/2 is placed on the horizontal table. The space between the lens and the mirror is filled with water of refractive index 4/3. It is found that when a point object is placed 15 cm above the lens, on the principal axis, the image coincide with the object itself. On representing with another liquid, the image and the object coincide at a distance 25 cm form the lens. Calculate the refractive index of the liquid.

[Ans : 1.6]

4. Consider the situation, where a point object is placed in air at a certain distance on the principal axis of the convex refracting surface. Refractive index of the surface is 3/2. If the other surface of the transparent medium is polished like a plane mirror. Calculate the distance of the object from the surface so that, light after refraction and reflection forms the image on the object itself Radius of curvature of the curved surface is 10 cm.

[Ans : 20 cm]

YOU ASKED WE ANSWERED

Do you have a question that you just can't get answered?

Use the vast expertise of our mtg team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

Q1. What causes thunder, and why can the sounds of thunder be anything from a nerve-racking clap to a drawn-out roll?

– Anjali Singh (New Delhi)

Ans. The primary source of sound in thunder is the shock wave produced by the lightning, which is an electric discharge. The lightning's huge current runs between cloud and ground (or between cloud and cloud) in a narrow channel with a radius of only a few centimetres. Within the channel, electrons are removed from air molecules by the huge electric field set up by charges on the ground and in the cloud. The freed electrons are then accelerated by the electric field and collide with air molecules, transferring their acquired energy to molecules. Because the gas of these molecules is then quite hot (the temperature may be 30,000 K), the gas expands. This process occurs so quickly that the channel of hot gas initially expands much faster than speed of sound, which sends a shock wave of abrupt pressure variations into the surrounding air, producing the sound of thunder.

If you are standing close to the lighting, you hear a terrifyingly loud crack as the shock wave sweeps past your ears. If you are farther away, you first hear sound from the nearest part of the strike and then you hear sound from portions of the strike that are higher or father from you. However, because the sound waves have spread out, this delayed sound may not be loud enough to terrify you. You probably will also hear reflections of the sound off hills, buildings, ground, and even clouds. These effects draw out the thunder into a roll.

You might hear a musical note if you are near lighting that consists of several rapid discharges. If the sound pulses from the discharges are closely spaced, you don't perceive them individually but instead perceive them as being part of a note. For example, if the time between successive pulses is 0.001 second, then you perceive sound with a frequency of 1/0.001 = 1000 hertz.

If the lighting is more than about 20 kilometres from you, may not hear any thunder. As the sound travels through the air, it is refracted (its path is bent) by changes in air temperature (hotter air is less dense than cooler air, and such density changes can bend the path taken by the sound). Because the air is typically cooler at cloud level than at ground level, the sound traveling towards you from distant lightning is bent up and away from you. However, in some thunderstorms, the air near the ground happens to be cooler than the higher air, a situation called a temperature inversion. During an inversion, sound from a lightning strike that is initially headed upward can be bent down. Worse, sometimes the sound emitted from various parts of a lightning strike can be focused (concentrated) in your direction.

Q2 Why are ice cubes cloudy, and is there any way to make clear (transparent) ice cubes?

– Neha Gupta (UP)

Ans. Ice is cloudy because light scatters from structures and materials inside the ice. Some of the material can be impurities that are concentrated by the freezing process. For example, as it advances into the water, the freezing process drives impurities into the liquid next to the interface of the liquid and the ice and forces dissolved air to form bubbles there. As the freezing advances and drives more air into these bubbles, the bubbles become longer and surrounded by ice. Thus, long wormholes (hollows) extend toward the centre of the cube.

The liquid-ice interface can advance only by conducting thermal energy from the interface to the surface of the cube, where it can be removed by cold air. The distance required of this conduction grows longer as the interface advances and so the advance slows. Thus, wormholes are typically wider near the centre of a cube (where the advance is slow) than near the surface (where the advance is faster). Some wormholes vary in radius because the freezer cycles between being on and off. Ice made from a brine (salty) solution can show more complex air bubbles that made form tap water. Under ideal conditions, they can form tiny spirals or zigzag patterns.

To make clear ice, distilled water can be used to avoid impurities and the water can be boiled for about 15 minutes to eliminate most of the dissolved air.

Poonam Shah (MP)

Q3. When the smoke rising from a campfire is viewed against a dark background such as surrounding trees, it is tinted blue, but when it is seen against a bright background such as the sky, it is tinted yellow, red, or orange. Why do the colours differ in the two viewing situations? – *Sreejit (Kerala)*

Ans. The smoke particles from the campfire are small enough that they scatter the blue end of the spectrum more strongly than the red end. So, the light continuing in the original direction weakens in blue and becomes yellow, red, or orange. If you view the smoke against a dark background, the light source must be behind you (it might be the Sun or a bright portion of the sky). Blue light scatters in your direction. If you view the smoke against a bright sky, that portion of the sky is the light source. You then intercept light that has become depleted in blue after passing through the smoke, thus the light is dominated by the red end of the spectrum.

Q4. When an automobile driver steps on the accelerator, the nose of the car moves upward. When the driver brakes, the nose moves downward. Why do these effects occur? – *Rajender (Pune)*

Ans. When the driver steps on the accelerator, there is an increased force on the tires from the roadway. This force is parallel to the roadway and directed toward the front of the automobile. This force provides a torque that tends to cause the car to rotate in the clockwise direction around the centre of mass of the car. The result of this rotation is a "nosing up" the car.

When the driver steps on the brake, there is an increased force on the tires from the roadway, directed toward the rear of the automobile. This force results in a torque that causes a counterclockwise rotation and the subsequent "nosing down" of the atomobile.

PHYSICS MUSING

Contd. from page no. 82

7. **(c) :** By law of conservation of linear momentum

$mu = mv + MV$ ...(i)

where m = mass of bullet
M = mass of block
u = velocity of bullet before collision
v = velocity of bullet after collision
V = velocity of block after collision

By law of conservation of energy

$$Mgh = \frac{1}{2}MV^2$$

$$\Rightarrow \quad V = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.1}$$

$$\therefore \quad V = 1.4 \text{ m s}^{-1}$$

Puting in equation (i), we get

$$500 \times 0.01 = 0.01v + 2(1.4)$$

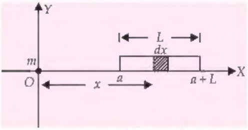
$$\therefore \quad v = \frac{2.2}{0.01} \text{ m s}^{-1} = 220 \text{ m s}^{-1}.$$

8. **(c) :** Loss in kinetic energy during head on collision $= \frac{1}{2}\left(\frac{mM}{m+M}\right)v^2$

$$\therefore \quad \frac{1}{2}\left(\frac{mM}{M+m}\right)v^2 = \Delta E$$

$$\Rightarrow \quad v = \sqrt{\frac{2(M+m)\Delta E}{Mm}}.$$

9. **(b):** Mass per unit length of the rod $= A + Bx^2$.
So the mass of length dx is $dM = dx(A + Bx^2)$



$$dF = \frac{Gm(dM)}{x^2} = Gm\left[\frac{A+Bx^2}{x^2}\right]dx$$

$$F = \int_a^{a+L} Gm\left(\frac{1}{x^2}\right)dx\left(A+Bx^2\right)$$

$$= \int_a^{a+L} Gm\left(\frac{A}{x^2}+B\right)dx = Gm\left[\frac{A}{a} - \frac{A}{a+L} + BL\right]$$

10. **(c) :** Total energy = kinetic energy + potential energy

or $\quad E_0 = \frac{1}{2}mv^2 - \frac{GMm}{r}$ ...(i)

Further, $\frac{mv^2}{r} = \frac{GMm}{r^2}$

or $\quad \frac{1}{2}mv^2 = \frac{GMm}{2r}$ ...(ii)

Substituting the value of $\frac{1}{2}mv^2$ in equation (i) from equation (ii), we get

$$E_0 = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$$

Therefore, potential energy $= -\frac{GMm}{r} = 2E_0$

 Contd. from page no. 26

$$= 4R\left(\frac{4-1}{4}\right) = 3R \Rightarrow \lambda_{2\to1} = \frac{1}{3R}$$

For $n = 3 \to n = 1$

$$\frac{1}{\lambda_{3\to1}} = R(2)^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$$= 4R\left(\frac{1}{1} - \frac{1}{9}\right) = 4R\left(\frac{9-1}{9}\right) = \frac{32R}{9}$$

$$\Rightarrow \lambda_{3\to1} = \frac{9}{32R}$$

51. (a) : The wavelengths of the spectral lines in the Balmer series is given by

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \quad n = 3, 4, 5, 6 .....$$

For hydrogen atom, $Z = 1$, $n = 3$ for first spectral line

$$\therefore \frac{1}{\lambda_H} = R(1)^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = R\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$= R\left[\frac{5}{36}\right] \quad ...(i)$$

For He^+ ion, $Z = 2$, $n = 4$ for second spectral line

$$\therefore \frac{1}{\lambda_{He^+}} = R(2)^2\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = 4R\left[\frac{1}{4} - \frac{1}{16}\right]$$

$$= 4R\left[\frac{3}{16}\right] \quad ...(ii)$$

Divide (i) by (ii), we get

$$\frac{\lambda_{He^+}}{\lambda_H} = \frac{5}{27}$$

$$\lambda_{He^+} = \lambda_H \times \frac{5}{27} = (6561\text{ Å}) \times \frac{5}{27} = 1215\text{ Å}$$

52. (4) : Decay constant, $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{1386\text{ s}} = 5 \times 10^{-4}\text{ s}^{-1}$

According to radioactive decay, $N = N_0 e^{-\lambda t}$

$$\frac{N}{N_0} = e^{-5\times10^{-4}\times80} \text{ or } \frac{N}{N_0} = e^{-0.04}$$

Fraction of nuclei decayed $= \frac{N_0 - N}{N_0} = 1 - \frac{N}{N_0}$

$$= 1 - e^{-0.04} = 1 - 0.96$$

$$= 0.04 = 4\%$$

53. (a) : The alpha decay of ${}^{210}_{84}Po$ is given by

$${}^{210}_{84}Po \to {}^{206}_{82}Pb + {}^{4}_{2}He$$

The energy released during this process is

$$Q = (M_{Po} - M_{Pb} - M_{He})c^2$$

$$= (209.982876 - 205.974455 - 4.002603)\text{ u} \times c^2$$

$$= (0.005818\text{ u})c^2$$

$$= (0.005818\text{ u}) \times 932\text{ MeV}$$

$$= 5.422\text{ MeV} = 5422\text{ keV}$$

Kinetic energy of α particle, $K_\alpha = \frac{(A-4)Q}{A}$

$$K_\alpha = \frac{(210-4)}{210} \times 5422\text{ keV} = \frac{206}{210} \times 5422\text{ keV}$$

$$= 5319\text{ keV}$$

54. (c) : (a) ${}^{6}_{3}Li \to {}^{2}_{1}H + {}^{4}_{2}He$

$m({}^{2}_{1}H) + m({}^{4}_{2}He) = 2.014102\text{ u} + 4.002603\text{ u}$
$= 6.016705\text{ u}$

$m({}^{6}_{3}Li) = 6.015123\text{ u}$

$\because \quad m_1 + m_2 > M$

So reaction is not possible.

Hence, (a) is an incorrect statement.

(b) ${}^{210}_{84}Po \to {}^{209}_{83}Bi + {}^{1}_{1}H$

$m({}^{209}_{83}Bi) + m({}^{1}_{1}H) = 208.980388\text{ u} + 1.007825\text{ u}$
$= 209.988213\text{ u}$

$m({}^{210}_{84}Po) = 209.982876\text{ u} \quad \because \quad m_1 + m_2 > M$

So reaction is not possible.

Hence, (b) is an incorrect statement.

(c) ${}^{2}_{1}H + {}^{4}_{2}He \to {}^{6}_{3}Li$

$m({}^{2}_{1}H) + m({}^{4}_{2}He) = 2.014102\text{ u} + 4.002603\text{ u}$
$= 6.016705\text{ u}$

$m({}^{6}_{3}Li) = 6.015123\text{ u}$

$m_3 + m_4 > M'$

$\therefore$ Deuteron and alpha particle can go complete fusion.

Hence, (c) is a correct statement.

(d) ${}^{70}_{30}Zn + {}^{82}_{34}Se \to {}^{152}_{64}Gd$

$m({}^{70}_{30}Zn) + m({}^{82}_{34}Se) = 69.925325\text{ u} + 81.916709\text{ u}$
$= 151.842034\text{ u}$

$m({}^{152}_{64}Gd) = 151.919803\text{ u}$

$\because \quad m_3 + m_4 < M'$

So, reaction is not possible.

Hence, (d) is a wrong statement.

55. (c) : In alpha decay, atomic number decreases by 2 and mass number decreases by 4. $P \to 2$

In β^+ decay, atomic number decreases by 1 and mass number remains unchanged. $Q \to 1$

In fission, heavy nucleus breaks into two light nuclei. $R \to 4$

In proton emission, both atomic number and mass number decreases by 1. $S \to 3$

56. (d)

57. (c) : $K_p + K_{\bar{e}} + K_{\bar{\nu}} = 0.8 \times 10^6\text{ eV}$

When electron has zero kinetic energy, energy is

shared by antineutrino and proton.

Then, $K_p + K_{\bar{\nu}} = 0.8 \times 10^6$ eV

As antineutrino is very light mass in comparison to proton so it will have almost contribution in total energy.

$\therefore$ Its energy is almost 0.8×10^6 eV

58. (1) : Activity, $A = \lambda N$ or $A = \frac{1}{\tau}N \quad \left(\text{As} \quad \lambda = \frac{1}{\tau}\right)$

where τ is the mean life time

$N = A\tau = (10^{10}\ \text{decay/s})(10^9\ \text{s})$

$= 10^{19}$ atoms

Mass of the sample, $m = N \times$ (mass of 1 atom)

$= 10^{19} \times 10^{-25}$ kg = 1 mg

59. (b): 1 MSD = 5.15 cm – 5.10 cm = 0.05 cm

50 VSD = 2.45 cm

$$1\ \text{VSD} = \frac{2.45}{50}\ \text{cm} = 0.049\ \text{cm}$$

Least count of vernier, LC = 1 MSD – 1 VSD

= 0.05 cm – 0.049 cm

= 0.001 cm

Diameter of the cylinder

= Main scale reading

+ Vernier scale reading × least count

= 5.10 + (24)(0.001) = 5.124 cm

60. (a) : Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{Total no. of circular divisions on circular scale}}$$

$$= \frac{0.5\ \text{mm}}{100} = 0.005\ \text{mm}$$

Least count of micrometer

$$= \frac{\text{Pitch}}{\text{Total no. of circular divisions on circular scale}}$$

$$= \frac{0.5\ \text{mm}}{100} = 0.005\ \text{mm}$$

The errors in both l and d are least count errors.

$\therefore \quad \Delta l = \Delta d = 0.005$ mm

$$Y = \frac{4MLg}{\pi l d^2}$$

The maximum probable error in Y is

$$\frac{\Delta Y}{Y} = \frac{\Delta l}{l} + \frac{2\Delta d}{d}$$

Error due to l measurement,

$$\frac{\Delta l}{l} = \frac{0.005\ \text{mm}}{0.25\ \text{mm}} = 0.02$$

Error due to d measurement,

$$\frac{2\Delta d}{d} = \frac{2 \times 0.005\ \text{m}}{0.5\ \text{mm}} = 0.02$$

So error in Y due to measurement of l is same that of due to measurement of d.

61. (c) : Least count of screw gauge

$$= \frac{\text{Pitch}}{\text{No. of divisions on the circular scale}}$$

$$= \frac{0.5\ \text{mm}}{50} = 0.01\ \text{mm}$$

Diameter of ball, D = MSR + CSR × LC

= 2.5 mm + (20)(0.01 mm)

= 2.7 mm

$$\text{Density, } \rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{\frac{4\pi}{3}\left(\frac{D}{2}\right)^3}$$

The relative error in the density is

$$\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + \frac{3\Delta D}{D}$$

The relative percentage in the density is

$$\frac{\Delta\rho}{\rho} \times 100 = \left[\frac{\Delta M}{M} + \frac{3\Delta D}{D}\right] \times 100$$

$$= 2\% + \frac{3 \times 0.01}{2.7} \times 100$$

$$= 2\% + 1.11\% = 3.11\%$$